CHAPTER 10 VECTORS

Syllabus

Vectors and Scalars, Magnitude and direction of a vector. Direction cosines and direction ratios of a vector , Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

In this chapter you will study

- *Some basic concepts about vectors*
- *different types of vectors*
- *various operations on vectors*
- *using concepts of vectors, finding area of triangle and parallelogram*
- *dot product of vectors*
- *cross product of vectors*

List of Topics

Topic-1: Basic Algebra of Vectors **Page No. 241**

Topic-2: Dot Product of Vectors **Page No. 248**

Topic-3: Cross Product **Page No. 255**

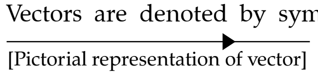
Topic-1 Basic Algebra of Vectors

Concepts Covered • *Basic concepts of vectors,* • *Operations on vectors* • *Different types of vectors,* • *Triangle Law,* • *Parallelogram Law*

Revision Notes

1. Vector : Basic Introduction :

- A physical quantity having **magnitude** as well as the direction is called a vector. It is denoted as $\vec{AB}$ or $\vec{a}$. Its magnitude (or modulus) is $|\vec{AB}|$ or $|\vec{a}|$ otherwise, simply AB or a.
- Vectors are denoted by symbols such as $\vec{a}$.



[Pictorial representation of vector]

2. Initial and Terminal Points :

The initial and terminal points means that point from which the vector originates and terminates respectively.

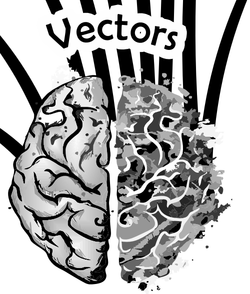
Key Words

Magnitude: It is defined as the maximum extent of size and the direction of an object. Magnitude is used as a common factor in vector and scalar quantities.

3. Position Vector :

The position vector of a point say $P(x, y, z)$ is $\vec{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and the magnitude is $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.

Vectors



Unit vector

For a given vector $\vec{a}$, the vector $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$ gives the unit vector in the direction of $\vec{a}$. *eg*, if $\vec{a} = 5\hat{i}$, then $\hat{a} = \dfrac{5\hat{i}}{|5|} = \hat{i}$, which is a unit vector.

Direction ratios and direction cosines

The scalar components of a vector are its direction ratios, and represent its projections along the respective axes. The magnitude *(r)* direction ratios *(a,b,c)* and direction cosines *(l,m,n)* of vector $a\hat{i} + b\hat{j} + c\hat{k}$ are related as:

$l = \dfrac{a}{r}, m = \dfrac{b}{r}, n = \dfrac{c}{r}$

eg : If $\overrightarrow{AB} = \hat{i} + 2\hat{j} + 3\hat{k}$, then $r = \sqrt{1+4+9} = \sqrt{14}$

Direction ratios are (1,2,3) and direction cosines are $\left(\dfrac{1}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}}\right)$

Position Vector

The position vector of a point R dividing a line segment joining *P,Q* whose position vectors are $\vec{a}, \vec{b}$ resp., in the ratio $m : n$

(i) internally is $\dfrac{n\vec{a} + m\vec{b}}{m+n}$, (ii) externally is $\dfrac{m\vec{b} - n\vec{a}}{m-n}$

Vector

A quantity that has both magnitude and direction is called a vector. The distance between the initial and terminal points of a vector is called its magnitude. Magnitude of vector $\overrightarrow{AB}$ is $|\overrightarrow{AB}|$.

Scalar product of two vectors

If $\vec{a}, \vec{b}$ are the vectors and 'θ', angle between them, then their scalar product $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow \cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

Cross product of two vectors

$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$, $\hat{n}$ is a unit vector perpendicular to line joining *a,b*.

Position of vectors

Position vector of a point *P (x, y, z)* is $x\hat{i} + y\hat{j} + z\hat{k}$ and its magnitude is $OP(r) = \sqrt{x^2 + y^2 + z^2}$. eg: Position vector of *P (2,3,5)* is $2\hat{i} + 3\hat{j} + 5\hat{k}$ and its magnitude is $\sqrt{2^2 + 3^2 + 5^2} = \sqrt{38}$.

Types of Vectors

(i) Zero vector (initial and terminal points coincide)
(ii) Unit vector (magnitude is unity)
(iii) Coinitial vectors (same initial points)
(iv) Collinear vectors (parallel to the same line)
(v) Equal vectors (same magnitude and direction)
(vi) Negative of a vector (same magnitude, opp. direction)

If we have two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and λ is any scalar, then-

(i) $\vec{a} \pm \vec{b} = (a_1 \pm b_1)\hat{i} + (a_2 \pm b_2)\hat{j} + (a_3 \pm b_3)\hat{k}$

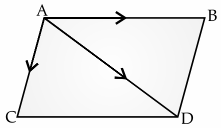
(ii) $\lambda\vec{a} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$

(iii) $\vec{a}.\vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ and

(iv) $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Properties of Vector

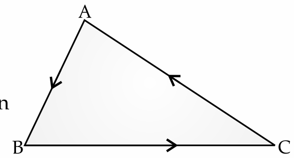
The vector sum of two co-initials vectors is given by the diagonal of the parallelogram whose adjacent sides are given vectors.



if $\overrightarrow{AB}, \overrightarrow{AC}$ are the given vectors, then $\overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AD}$

The vector sum of the three sides of a triangle taken in order is $\vec{0}$ *.i.e*

if *ABC* is given triangle, then $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$.



Trace the Mind Map

- First Level
- Second Level
- Third Level

The vector $\overrightarrow{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is said to be in its **component form**. Here x, y, z are called the scalar components or rectangular components of $\vec{r}$ and $x\hat{i}$, $y\hat{j}$, $z\hat{k}$ are the vector components of $\vec{r}$ along X, Y, Z-axis respectively.

- Also, $\overrightarrow{AB}$ = (Position Vector of B) – (Position Vector of A). For example, let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$. Then, $\overrightarrow{AB}$ = $(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$.
- Here $\hat{i}, \hat{j}$ and $\hat{k}$ are the unit vectors along the axes OX, OY and OZ respectively (The discussion about unit vectors is given later under 'types of vectors').

4. Direction Ratios and Direction Cosines:

If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then coefficient of $\hat{i}, \hat{j}, \hat{k}$ in $\vec{r}$ *i.e.*, x, y, z are called the direction ratios (abbreviated as d.r.'s) of vector $\vec{r}$. These are denoted by a, b, c (*i.e.*, $a = x$, $b = y$, $c = z$; in a manner we can say that scalar components of vector $\vec{r}$ and its d.r.'s both are the same).

Also, the coefficients of $\hat{i}, \hat{j}, \hat{k}$ in $\hat{r}$ (which is the unit vector of $\vec{r}$) *i.e.*, $\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}$ are called direction cosines (which is abbreviated as d.c.'s) of vector $\vec{r}$.

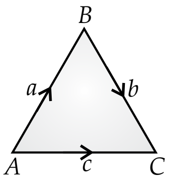
- These direction cosines are denoted by l, m, n such that $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$ and $l^2 + m^2 + n^2 = 1 \Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.
- It can be easily concluded that $\frac{x}{r} = l = \cos\alpha$, $\frac{y}{r} = m = \cos\beta$, $\frac{z}{r} = n = \cos\gamma$.
  Therefore,
  $\vec{r} = lr\hat{i} + mr\hat{j} + nr\hat{k} = r(\cos\alpha\hat{i} + \cos\beta\hat{j} + \cos\gamma\hat{k})$.
  [Here $r = |\vec{r}|$].

5. Addition of vectors

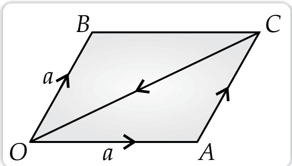
(a) Triangular law : If two adjacent sides (say sides AB and BC) of a triangle ABC are represented by $\vec{a}$ and $\vec{b}$ taken in same order, then the third side of the triangle taken in the reverse order gives the sum of vectors $\vec{a}$ and $\vec{b}$ *i.e.*,

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} \Rightarrow \overrightarrow{AC} = \vec{a} + \vec{b}$.

- Also since $\overrightarrow{AC} = -\overrightarrow{CA} \Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$.



(b) Parallelogram law : If two vectors $\vec{a}$ and $\vec{b}$ are represented in magnitude and the direction by the two adjacent sides (say OA and OB) of a parallelogram $OACB$, then their sum is given by that diagonal of parallelogram which is co-initial with $\vec{a}$ and $\vec{b}$ *i.e.*, $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB}$.



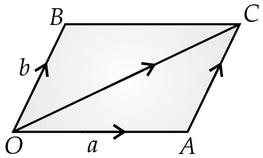
6. Properties of Vector Addition

(a) Commutative property : $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

Consider $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ be any two given vectors, then

$\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k} = \vec{b} + \vec{a}$.

(b) Associative property : $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$.



(c) Additive identity property : $\vec{a} + \vec{0} = \vec{0} + \vec{a} = \vec{a}$.

(d) Additive inverse property :

$\vec{a} + (-\vec{a}) = \vec{0} = (-\vec{a}) + \vec{a}$.

Note : Multiplication of a vector by a scalar

Let $\vec{a}$ be any vector and k be any non-zero scalar. Then the product $k\vec{a}$ is defined as a vector whose magnitude is $|k|$ times that of $\vec{a}$ and the direction is

(i) same as that of $\vec{a}$ if k is positive, and

(ii) opposite as that of $\vec{a}$ if k is negative.

Amazing Facts

- In biology, a vector is a living organism that transmits an infections agent from an infected animal a human or another animal. Vectors are frequency arthropods, such as mosquitoes, ticks, files, fleas an lice.
- In video games, we use vectors to represent the velocity of players, but also to control where they are aiming or what they can see (where they are facing).

Key Fact

- Vector calculus and its sub objective vector fields was invented by two men J. Willard Gibbs and Oliver Heaviside at the end of 19^{th} century.

Key Fact

- Vectors can be placed in a new position without rotating it. It still has the same magnitude and direction, and is identical to the vector at the beginning.

Know the Terms

Types of Vectors :

(a) Zero or Null vector : It is that vector whose initial and terminal points are coincident. It is denoted by $\vec{0}$. of course its magnitude is 0 (zero).

- Any non-zero vector is called a **proper vector**.

(b) Co-initial vectors : Those vectors (two or more) having the same starting point are called the co-initial vectors.

(c) Co-terminus vectors : Those vectors (two or more) having the same terminal point are called the co-terminus vectors.

(d) Negative of a vector : The vector which has the same magnitude as the $\vec{r}$ but opposite direction. It is denoted by $-\vec{r}$. Hence if, $\vec{AB}=\vec{r}$ or $\vec{BA}=-\vec{r}$ *i.e.*, $\vec{AB}=-\vec{BA}$, $\vec{PQ}=-\vec{QP}$ etc.

(e) Unit vector : It is a vector with the unit magnitude.

The unit vector in the direction of vector $\vec{r}$ is given by $\hat{r}=\frac{\vec{r}}{|\vec{r}|}$ such that $|\hat{r}|=1$, so, if $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ then its unit vector is :

$$\hat{r}=\frac{x}{\sqrt{x^2+y^2+z^2}}\hat{i}+\frac{y}{\sqrt{x^2+y^2+z^2}}\hat{j}+\frac{z}{\sqrt{x^2+y^2+z^2}}\hat{k}.$$

- Unit vector perpendicular to the plane $\vec{a}$ and $\vec{b}$ is : $\pm\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$.

(f) Reciprocal of a vector : It is a vector which has the same direction as the vector $\vec{r}$ but magnitude equal to the reciprocal of the magnitude of $\vec{r}$. It is denoted as $\vec{r}^{\,-1}$. Hence $\left|\vec{r}^{\,-1}\right|=\frac{1}{|\vec{r}|}$.

(g) Equal vectors : Two vectors are said to be equal if they have the same magnitude as well as direction, regardless of the position of their initial points.

Thus $\vec{a}=\vec{b}\Leftrightarrow\begin{cases}|\vec{a}|=|\vec{b}|\\ \vec{a}\text{ and }\vec{b}\text{ have same direction}\end{cases}$

Also, if $\vec{a}=\vec{b}\Rightarrow a_1\hat{i}+a_2\hat{j}+a_3\hat{k}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\Rightarrow a_1=b_1,\ a_2=b_2, a_3=b_3.$

(h) Collinear or Parallel vector : Two vectors $\vec{a}$ and $\vec{b}$ are collinear or parallel if there exists a non-zero scalar λ such that $\vec{a}=\lambda\vec{b}$.

- It is important to note that the respective coefficients of $\hat{i},\hat{j},\hat{k}$ in $\vec{a}$ and $\vec{b}$ are proportional provided they are parallel or collinear to each other.
- The d.r's of parallel vectors are same (or are in proportion).
- The vectors $\vec{a}$ and $\vec{b}$ will have same or opposite direction as λ is positive or negative respectively.
- The vectors $\vec{a}$ and $\vec{b}$ are collinear if $\vec{a}\times\vec{b}=\vec{0}$.

(i) Free vectors : The vectors which can undergo parallel displacement without changing its magnitude and direction are called free vectors.

Key Formulae

The position vector of a point say *P* dividing a line segment joining the points *A* and *B* whose position vectors are $\vec{a}$ and $\vec{b}$ respectively, in the ratio *m* : *n*.

(a) Internally, $\vec{OP}=\frac{m\vec{b}+n\vec{a}}{m+n}$

(b) Externally, $\vec{OP}=\frac{m\vec{b}-n\vec{a}}{m-n}$

- Also if point *P* is the mid-point of line segment *AB*, then $\vec{OP}=\frac{\vec{a}+\vec{b}}{2}$.

Mnemonics

Types Of Vectors (A)

Collinear Unit

Teachers of **V**arious **N**ew **C**olleges **U**sing

Types Vectors Negative

Zoom **C**onferencing during **E**pidemic

Zero Coinitial Equal

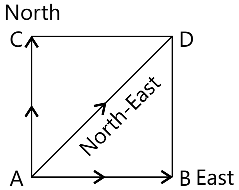
Interpretation :

Types of Vectors-

1. **Zero Vector -** Initial and terminal points coincide
2. **Unit Vector -** Magnitude is unity
3. **Coinitial Vectors -** Same initial points
4. **Collinear vectors -** Parallel to the same Line
5. **Equal Vectors -** Same magnitude and direction
6. **Negative of a vector -** Same magnitude, opp. direction

Properties Of Vectors(B)

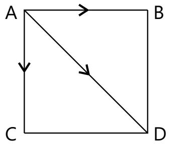
"Neither choose East nor choose north, always choose North-East and save your time".



Mnemonics

Interpretation :

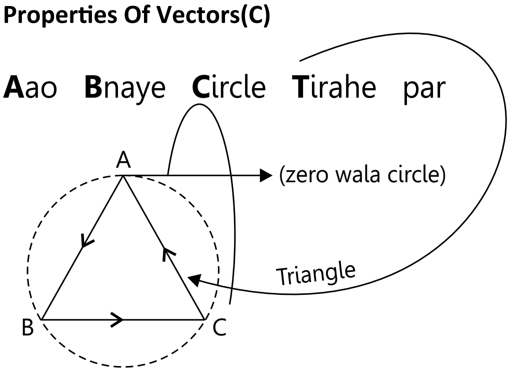
The vector sum of two coinitial vectors is given by the diagonal of the parallelogram whose adjacent sides are given vectors.



$\overrightarrow{AB} + \overrightarrow{AC} = \overrightarrow{AD}$

Properties Of Vectors(C)

Aao **B**naye **C**ircle **T**irahe par



Interpretation:

The vector sum of the three sides of a triangle taken in order is $\vec{O}$ i.e

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{O}$

OBJECTIVE TYPE QUESTIONS

A Multiple Choice Questions

Q. 1. If $\vec{a}$ is a non-zero vector of magnitude 'a' and λ a non-zero scalar, then $\lambda\vec{a}$ is unit vector if

(A) $\lambda = 1$ **(B)** $\lambda = -1$

(C) $a = |\lambda|$ **(D)** $a = \dfrac{1}{|\lambda|}$

Ans. Option (D) is correct.

***Explanation*:**

Vector $\lambda\vec{a}$ is a unit vector if

$$|\lambda\vec{a}| = 1$$

$$\Rightarrow |\lambda||\vec{a}| = 1$$

$$\Rightarrow |\vec{a}| = \frac{1}{|\lambda|} \qquad [\lambda \neq 0]$$

$$\Rightarrow a = \frac{1}{|\lambda|} \qquad [|\vec{a}| = a]$$

Therefore, vector $\lambda\vec{a}$ is a unit vector if $a = \dfrac{1}{|\lambda|}$.

Q. 2. What is if $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following is incorrect:

(A) $\vec{b} = \lambda\vec{a}$, for some scalar λ

(B) $\vec{a} = \pm\vec{b}$

(C) the respective components of $\vec{a}$ and $\vec{b}$ are proportional

(D) both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes

Ans. Option (D) is correct.

***Explanation*:**

If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then they are parallel.

Therefore, we have

$\vec{b} = \lambda\vec{a}$ (For some scalar λ)

If $\lambda = \pm 1$, then $\vec{a} = \pm\vec{b}$.

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then,

$$\vec{b} = \lambda\vec{a}$$

$$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = \lambda\left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)$$

$$\Rightarrow b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$$

$$\Rightarrow b_1 = \lambda a_1, b_2 = \lambda a_2, b_3 = \lambda a_3$$

$$\Rightarrow \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda$$

So that, the respective components of $\vec{a}$ and b are proportional. However, vectors $\vec{a}$ and $\vec{b}$ can have different directions.

Q. 3. What is the vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is

(A) $\hat{i} - 2\hat{j} + 2\hat{k}$ (B) $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$

(C) $3(\hat{i} - 2\hat{j} + 2\hat{k})$ (D) $9(\hat{i} - 2\hat{j} + 2\hat{k})$

Ans. Option (C) is correct.

***Explanation*:**

Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

Any vector in the direction of a vector $\vec{a}$ is given by

$$\frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{1^2 + 2^2 + 2^2}} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$$

$\therefore$ Vector in the direction of $\vec{a}$ with magnitude 9

$$= 9\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} = 3(\hat{i} - 2\hat{j} + 2\hat{k})$$

Q. 4. The position vector of the point which divides the join of points $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$ in the ratio 3 : 1 is

(A) $\frac{3\vec{a} - 2\vec{b}}{2}$ (B) $\frac{7\vec{a} - 8\vec{b}}{4}$

(C) $\frac{3\vec{a}}{4}$ (D) $\frac{5\vec{a}}{4}$

Ans. Option (D) is correct.

Explanation:

Let the position vector of the R divides the join of points $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$.

$\therefore$ Position vector, $R = \frac{3(\vec{a} + \vec{b}) + 1(2\vec{a} - 3\vec{b})}{3+1}$

Since, the position vector of a point R dividing the line segments joining the points P and Q, whose position vectors are $\vec{p}$ and $\vec{q}$ in the ratio $m : n$ internally, is given by $\frac{m\vec{q} + n\vec{p}}{m+n}$.

$$\therefore R = \frac{5\vec{a}}{4}$$

Q. 5. The vector having initial and terminal points as (2, 5, 0) and (–3, 7, 4), respectively is:

(A) $-\hat{i} + 12\hat{j} + 4\hat{k}$ (B) $5\hat{i} + 2\hat{j} - 4\hat{k}$

(C) $-5\hat{i} + 2\hat{j} + 4\hat{k}$ (D) $\hat{i} + \hat{j} + \hat{k}$

Ans. Option (C) is correct.

Explanation:

$$\text{Required vector} = (-3-2)\hat{i} + (7-5)\hat{j} + (4-0)\hat{k} = -5\hat{i} + 2\hat{j} + 4\hat{k}$$

Q. 6. The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$

(C) $\frac{5}{2}$ (D) $\frac{2}{5}$

Ans. Option (A) is correct.

Explanation:

Let $\vec{a} = 3\hat{i} - 6\hat{j} + \hat{k}$

and $\vec{b} = 2\hat{i} - 4\hat{j} + \lambda\hat{k}$

Since, $\vec{a} \parallel \vec{b}$

$\Rightarrow \frac{3}{2} = \frac{-6}{-4} = \frac{1}{\lambda}$

$\Rightarrow \lambda = \frac{2}{3}$

Q. 7. If $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$, then the range of $|\lambda\vec{a}|$ is

(A) [0, 8] (B) [–12, 8]

(C) [0, 12] (D) [8, 12]

Ans. Option (C) is correct.

Explanation:

We have, $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$

$\therefore |\lambda\vec{a}| = |\lambda||\vec{a}| = \lambda|4|$

at $\lambda = -3$

$\Rightarrow |\lambda\vec{a}| = |-3|4 = 12,$

at $\lambda = 0$

$|\lambda\vec{a}| = |0|4 = 0,$

and $|\lambda\vec{a}| = |2|4 = 8,$

at $\lambda = 2$

So, the range of $|\lambda\vec{a}|$ is [0,12].

Q. 8. Which of the following statement is true.

(A) $\vec{a}$ and $\overrightarrow{-a}$ are collinear

(B) Two collinear vectors are always equal in magnitude

(C) Two vectors having same magnitude are collinear

(D) Two collinear vectors having the same magnitude are equal

Ans. Option (A) is correct.

Explanation:

(A) True

Vectors $\vec{a}$ and $-\vec{a}$ are parallel to the same line.

(B) False

Collinear vectors are those vectors that are parallel to the same line.

(C) False

It is not necessary for two vectors having the same magnitude to be parallel to the same line.

(D) False

Two vectors are said to be equal if they have the same magnitude and direction, regardless of the positions of their initial points.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (1 mark each)

Q. 1. The position vectors of two points A and B are $\overline{OA} = 2\hat{i} - \hat{j} - \hat{k}$ and $\overline{OB} = 2\hat{i} - \hat{j} + 2\hat{k}$, respectively. The position vectors of a point which divides the line segment joining A and B in the ratio 2 : 1 is ....

Topper Answer, 2020

Sol.

2 1

PV of A $(2\hat{i}-\hat{j}-\hat{k})$ P PV of B $(2\hat{i}-\hat{j}+2\hat{k})$

PV of P $= \dfrac{2(2\hat{i}-\hat{j}+2\hat{k})+(2\hat{i}-\hat{j}-\hat{k})}{2+1}$

$= \dfrac{4\hat{i}-2\hat{j}+4\hat{k}+2\hat{i}-\hat{j}-\hat{k}}{3}$

$= 2\hat{i}-\hat{j}+\hat{k}$

Q. 2. Find a unit vector in the direction opposite to $-\frac{3}{4}\hat{j}$.

R [CBSE SQP 2020-21]

Q. 3. Give an example of vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = |\vec{b}|$ but $\vec{a} \neq \vec{b}$.

R&U [SQP 2017-18]

Q. 4. Write the number of vectors of unit length perpendicular to both the vector $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$.

A [O.D. Set I, II, III 2016]

Sol. There are two such vectors of unit length perpendicular to both the given vectors $\vec{a}$ and $\vec{b}$ and vectors are $\pm\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$.

Q. 5. Find the position vector of a point which divides the join of points with position vectors $(\vec{a}-2\vec{b})$ and $(2\vec{a}+\vec{b})$ externally in the ratio 2 : 1.

R&U [Delhi Set I, II, III 2016]

Sol. Required vector $= \dfrac{1(\vec{a}-2\vec{b})-2(2\vec{a}+\vec{b})}{1-2}$ ½

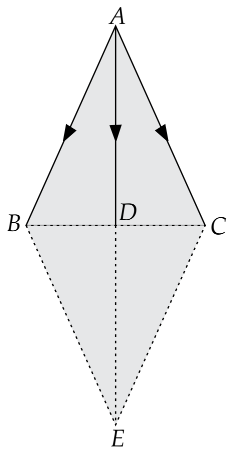
$= \dfrac{(\vec{a}-2\vec{b})-(4\vec{a}+2\vec{b})}{-1}$

$= 3\vec{a}+4\vec{b}$ ½

Q. 6. The two vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represent the two sides *AB* and *AC*, respectively of ΔABC. Find the length of the median through *A*.

AI A [Delhi Set I, II, III 2016] [Foreign 2015]

Sol. $\vec{AB} = \hat{j}+\hat{k}$ and $\vec{AC} = 3\hat{i}-\hat{j}+4\hat{k}$



Now *ABEC* represent a parallelogram with *AE* as the diagonal.

$\vec{AE} = \vec{AB}+\vec{AC}$ ½

$= (\hat{j}+\hat{k})+(3\hat{i}-\hat{j}+4\hat{k}) = 3\hat{i}+5\hat{k}$

Now, $|\vec{AE}| = \left|\sqrt{(3)^2+(5)^2}\right| = \sqrt{9+25}$

$= \sqrt{34}$ units

$\therefore$ $|\vec{AD}| = \dfrac{1}{2}\sqrt{34}$ units ½

Q. 7. **If $\vec{a}$ and $\vec{b}$ denote the position vectors of points *A* and *B* respectively and *C* is a point on *AB* such that *AC* = 2 *CB*, then write the position vector of *C*.**

A I R&U [Outside Delhi Set I, II, III Comptt. 2016]

Sol. $AC : CB = 2:1$

Position vector of C

$$= \frac{\vec{a}+2\vec{b}}{3} \qquad 1$$

[CBSE Marking Scheme 2016]

Q. 8. **Find a vector in the direction of $\vec{a} = \hat{i} - 2\hat{j}$ that has magnitude 7 units.** R&U [NCERT]

[Delhi Set I, II, III Comptt. 2015]

Sol.
$$\hat{a} = \frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j} = \frac{\vec{a}}{|\vec{a}|} \qquad ½$$

then
$$7\hat{a} = \frac{7}{\sqrt{5}}\hat{i} - \frac{14}{\sqrt{5}}\hat{j} \qquad ½$$

[CBSE Marking Scheme 2015]

Short Answer Type Questions-I (2 marks each)

Q. 1. **Shown below are two vectors in their component forms.**

$\vec{u} = 3\hat{i} - p\hat{j} + 5\hat{k}$

$\vec{v} = -6\hat{i} + 14\hat{j} + q\hat{k}$

For what values of *p* and *q* are the vectors collinear? Show your steps.

A I [CBSE Practice Questions 2022]

Sol. Here, $\vec{u}$ is parallel to $\vec{v}$.

We know that if two vectors $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{j}$ are parallel, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, we have

$$\frac{3}{-6} = \frac{-p}{14} = \frac{5}{q} \qquad 1$$

$$\therefore \quad \frac{3}{-6} = \frac{-p}{14}$$

$$\Rightarrow \quad p = 7 \qquad ½$$

and
$$\frac{3}{-6} = \frac{5}{q}$$

$$\Rightarrow \quad q = -10 \qquad ½$$

Thus, when $p = 7$ and $q = -10$, then vectors are collinear.

Q. 2. **Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$** R&U [SQP 2018-19]

Q. 3. **Find the angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$** A [SQP 2018-19]

Q. 4. **The position vectors of points *A*, *B* and *C* are $\lambda\hat{i} + 3\hat{j}$, $12\hat{i} + \mu\hat{j}$ and $11\hat{i} - 3\hat{j}$ respectively. If *C* divides the line segment joining *A* and *B* in the ratio 3 : 1, find the values of λ and μ.**

R&U [Delhi Comptt. 2017]

Sol.
$$11\hat{i} - 3\hat{j} = \frac{3(12\hat{i} + \mu\hat{j}) + 1(\lambda\hat{i} + 3\hat{j})}{4} \qquad 1$$

$$44 = 36 + \lambda, -12 = 3\mu + 3$$

$$\lambda = 8, \mu = -5 \qquad ½ + ½$$

[CBSE Marking Scheme, 2017]

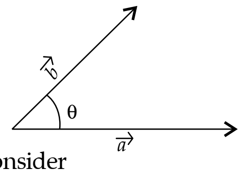
Topic-2 Dot Product of Vectors

Concepts Covered • *Properties of dot product,* • *Projection of a vector*

Revision Notes

1. Products of Two Vectors and Projection of Vectors

(a) Scalar Product or Dot Product : The dot product of two vectors $\vec{a}$ and $\vec{b}$ is defined by, $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$ where θ is the angle between $\vec{a}$ and $\vec{b}$, $0 \le \theta \le \pi$.



Consider

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} +$,

then $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.

➲ **Projection of a vector :** $\vec{a}$ on the other vector say $\vec{b}$ is given as $\left(\dfrac{\vec{a}.\vec{b}}{|\vec{b}|}\right)$.

➲ **Projection of a vector :** $\vec{b}$ on the other vector say $\vec{a}$ is given as $\left(\dfrac{\vec{a}.\vec{b}}{|\vec{a}|}\right)$.

Key Words

Projection: The image of a geometrical figure reproduced on a line, plane or surface.

Scalar: A physical quantity that is completely described by its magnitude.

Know the Properties (Dot Product)

- **Properties/Observations of Dot product**

➲ $\hat{i}.\hat{i} = |\hat{i}||\hat{i}|\cos 0 = 1$ or $\hat{i}.\hat{i} = 1 = \hat{j}.\hat{j} = \hat{k}\cdot\hat{k}$

➲ $\hat{i}.\hat{j} = |\hat{i}||\hat{j}|\cos\dfrac{\pi}{2} = 0$ or $\hat{i}.\hat{j} = 0 = \hat{j}.\hat{k} = \hat{k}\cdot\hat{i}$

➲ $\vec{a}\cdot\vec{b} \in R$, where R is real number *i.e.*, any scalar.

➲ $\vec{a}\cdot\vec{b} = \vec{b}.\vec{a}$ (Commutative property of dot product).

➲ $\vec{a}\cdot\vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$ or $|\vec{a}| = 0$ or $|\vec{b}| = 0$.

➲ If $\theta = 0$, then $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|$. Also $\vec{a}.\vec{a} = |\vec{a}|^2 = a^2$; as θ in this case is 0.

Moreover if $\theta = \pi$, then $\vec{a}\cdot\vec{b} = -|\vec{a}||\vec{b}|$.

➲ $\vec{a}.\left(\vec{b}+\vec{c}\right) = \vec{a}.\vec{b}+\vec{a}.\vec{c}$

(Distributive property of dot product).

➲ $\vec{a}.\left(-\vec{b}\right) = -\left(\vec{a}.\vec{b}\right) = \left(-\vec{a}\right).\vec{b}$.

Key Formulae

➲ Angle between two vectors $\vec{a}$ and $\vec{b}$ can be found by the expression given below :

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \text{ or, } \theta = \cos^{-1}\left(\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right).$$

OBJECTIVE TYPE QUESTIONS

A Multiple Choice Questions

Q. 1. If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then the value of λ is

(A) 0

(B) 1

(C) $-\dfrac{2}{3}$

(D) $-\dfrac{3}{2}$

Ans. Option (C) is correct.

Explanation:

Topper Answer, 2020

Sol.

$\Rightarrow \vec{a}\cdot\vec{b} = 0$

$2 + 3\lambda = 0$

$\Rightarrow \lambda = \dfrac{-2}{3}$

C. $-2/3$.

Q. 2. Let $\vec{a}$ and $\vec{b}$ be two-unit vectors and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if

(A) $\theta = \frac{\pi}{4}$ **(B)** $\theta = \frac{\pi}{3}$

(C) $\theta = \frac{\pi}{2}$ **(D)** $\theta = \frac{2\pi}{3}$

Ans. Option (D) is correct.

Explanation: Let $\vec{a}$ and $\vec{b}$ be two-unit vectors and θ be the angle between them.

Then, $|\vec{a}+\vec{b}| = |\vec{b}| = 1.$

Now, $\vec{a}+\vec{b}$ is a unit vector if

$$|\vec{a}+\vec{b}| = 1$$

$$\Rightarrow (\vec{a}+\vec{b})^2 = 1$$

$$\Rightarrow (\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = 1$$

$$\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}.\vec{b} = 1$$

$$\Rightarrow |\vec{a}|^2+2\vec{a}.\vec{b}+|\vec{b}|^2 = 1$$

$$\Rightarrow 1^2+2|\vec{a}||\vec{b}|\cos\theta+1^2 = 1$$

$$\Rightarrow \cos\theta = -\frac{1}{2}$$

$$\Rightarrow \theta = \frac{2\pi}{3}$$

So that, $|\vec{a}+\vec{b}|$ is a unit vector if $\theta = \frac{2\pi}{3}$.

Q. 3. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively, and $\vec{a}.\vec{b} = 2\sqrt{3}$ is:

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ **(D)** $\frac{5\pi}{2}$

Ans. Option (B) is correct.

Explanation:

Here, $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 4$ and $\vec{a}.\vec{b} = 2\sqrt{3}$ [Given]

We know that,

$$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\Rightarrow 2\sqrt{3} = \sqrt{3}.4.\cos\theta$$

$$\Rightarrow \cos\theta = \frac{2\sqrt{3}}{4\sqrt{3}} = \frac{1}{2}$$

$$\therefore \theta = \frac{\pi}{3}$$

Q. 4. Find the value of λ such that the vectors $\vec{a} = 2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+3\hat{k}$ are orthogonal.

(A) 0 **(B)** 1

(C) $\frac{3}{2}$ **(D)** $\frac{-5}{2}$

Ans. Option (D) is correct.

Explanation:

Since, two non-zero vectors $\vec{a}$ and $\vec{b}$ are orthogonal,

i.e., $\vec{a}.\vec{b} = 0$

$$\therefore (2\hat{i}+\lambda\hat{j}+\hat{k})\cdot(\hat{i}+2\hat{j}+3\hat{k}) = 0$$

$$\Rightarrow 2+2\lambda+3 = 0$$

$$\therefore \lambda = \frac{-5}{2}$$

Q. 5. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c} = \vec{0}$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is

(A) 1 **(B)** 3

(C) $\frac{-3}{2}$ **(D)** None of these

Ans. Option (C) is correct.

Explanation:

We have, $\vec{a}+\vec{b}+\vec{c} = 0$ and , $\vec{b}^2 = 1, \vec{c}^2 = 1$

$$\therefore (\vec{a}+\vec{b}+\vec{c})(\vec{a}+\vec{b}+\vec{c}) = 0$$

$$\Rightarrow \vec{a}^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{a}+\vec{b}^2+\vec{b}\cdot\vec{c}+ \vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}+\vec{c}^2 = 0$$

$$\Rightarrow \vec{a}^2+\vec{b}^2+\vec{c}^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$$

$[\because \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a},\ \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{b}$ and $\vec{c}\cdot\vec{a} = \vec{a}\cdot\vec{c}]$

$$\Rightarrow 1+1+1+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$$

$$\Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = -\frac{3}{2}$$

Q. 6. The projection vector of $\vec{a}$ on $\vec{b}$ is

(A) $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)\vec{b}$ **(B)** $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

(C) $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$ **(D)** $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\hat{b}$

Ans. Option (B) is correct.

Explanation: Projection vector of $\vec{a}$ on $\vec{b}$ is given by,

$$= \vec{a}\cdot\frac{\vec{b}}{|\vec{b}|}$$

$$= \left(\vec{a}\cdot\frac{\vec{b}}{|\vec{b}|}\right)$$

Q. 7. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ and $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $|\vec{c}| = 5$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is

(A) 0 **(B)** 1

(C) –19 **(D)** 38

Ans. Option (C) is correct.

Explanation :

Here, $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ and $\vec{a}^2 = 4$, $\vec{b}^2 = 9$, $\vec{c}^2 = 25$

$$\therefore (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c}) = \vec{0}$$

$$\Rightarrow \vec{a}^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{a}+\vec{b}^2+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}+ \vec{c}\cdot\vec{b}+\vec{c}^2 = \vec{0}$$

$$\Rightarrow \quad \vec{a}^2 + b^2 + \vec{c}^2 + 2(\vec{a}\cdot b + b\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$$

$$[\because\ \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}]$$

$$\Rightarrow \quad 4 + 9 + 25 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$$

$$\Rightarrow \quad \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = \frac{-38}{2}$$

$$= -19$$

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (1 mark each)

Q. 1. Find the magnitude of each of the vectors $\vec{a}$ and $\vec{b}$, having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$.

R&U [Delhi/O.D. 2018]

Q. 2. If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ for $\vec{a} - \sqrt{2}\,\vec{b}$ to be unit vectors?

R&U [Delhi Set I, II, III Comptt. 2015] [O.D. Set-II, 2016]

Topper Answer, 2016

Sol. $\bar{a} - \sqrt{2}\,\bar{b}$ is a unit vector

$|\bar{a} - \sqrt{2}\,\bar{b}| = 1$

$\therefore\ |\bar{a} - \sqrt{2}\,\bar{b}|^2 = 1$

$|\bar{a}|^2 + |\sqrt{2}\,\bar{b}|^2 - 2\sqrt{2}\,\bar{a}\cdot\bar{b} = 1$ $\quad (|\bar{a}| = |\bar{b}| = 1)$ $\quad$ (θ: Angle between vectors $\bar{a}$ & $\bar{b}$)

$1 + 2 - 2\sqrt{2}\,\bar{a}\cdot\bar{b} = 1$ $\quad \bar{a}\cdot\bar{b} = \frac{1}{\sqrt{2}}$ $\Rightarrow |\bar{a}||\bar{b}|\cos\theta = \frac{1}{\sqrt{2}}$ $\quad \therefore \boxed{\theta = 45^\circ}$ (Angle between $\bar{a}$ & $\bar{b}$ is 45°)

Q. 3. If $\hat{a}, \hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then find value of $|2\hat{a} + \hat{b} + \hat{c}|$.

R&U [All India 2015]

Sol. Given $\hat{a}, \hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, *i.e.*,

$$\hat{a}.\hat{b} = \hat{b}.\hat{c} = \hat{c}.\hat{a} = 0 \quad ...(i)$$

and

$$|\hat{a}| = |\hat{b}| = |\hat{c}| = 1 \quad ...(ii)$$

Now, $|2\hat{a} + \hat{b} + \hat{c}|^2 = (2\hat{a} + \hat{b} + \hat{c}).(2\hat{a} + \hat{b} + \hat{c})$

$$= 4(\hat{a}.\hat{a}) + 2(\hat{a}.\hat{b}) + 2(\hat{a}.\hat{c}) + 2(\hat{b}.\hat{a})$$

$$+ (\hat{b}.\hat{b}) + (\hat{b}.\hat{c}) + 2(\hat{c}.\hat{a}) + (\hat{c}.\hat{b}) + (\hat{c}.\hat{c})$$

$$[\because \vec{a}.\vec{a} = |\vec{a}|^2]$$

[$\because$ Dot product is distributive over addition] ½

$$= 4(|\hat{a}|^2) + 2(0) + 2(0) + 2(0) + |\hat{b}|^2 + (0)$$

$$+ 2(0) + (0) + |\hat{c}|^2$$

[from Eq. (i) and $\vec{a}.\vec{b} = \vec{b}.\vec{a}$]

$$= 4(1) + 1 + 1 = 4 + 1 + 1 = 6$$

$$\therefore \quad |2\hat{a} + \hat{b} + \hat{c}| = \sqrt{6} \quad ½$$

[$\because$ length cannot be negative]

[CBSE Marking Scheme 2015]

Q. 4. Find the projection of vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$. R&U [NCERT]

[O.D. Set I, II, III Comptt. 2015]

Short Answer Type Questions-I (2 marks each)

Q. 1. If $\hat{a}$ and $\hat{b}$ are unit vectors, then prove that $|\hat{a} + \hat{b}| = 2\cos\frac{\theta}{2}$, where θ is the angle between them. A I [SQP 2021-22]

Sol.

$$(\hat{a}+\hat{b}).(\hat{a}+\hat{b}) = |\hat{a}|^2+|\hat{b}|^2+2(\hat{a}.\hat{b})$$

$$|\hat{a}+\hat{b}|^2 = 1 + 1 + 2\cos\theta \quad \mathbf{1}$$

$$= 2(1 + \cos\theta)$$

$$= 4\cos^2\frac{\theta}{2} \quad ½$$

$$\therefore \quad |\hat{a}+\hat{b}| = 2\cos\frac{\theta}{2} \quad ½$$

[CBSE Marking Scheme, 2022]

Q. 2. Find $|\vec{a}|$ and $|\vec{b}|$, if $|\vec{a}| = 2|\vec{b}|$ and $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 12$.

R **[CBSE OD Set-I, II, III 2020]**

Q. 3. If $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$.

R&U **[CBSE SQP, 2020]**

Sol. $(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c}) = 0$

$$\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{a}+\vec{b}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$$
$$+\vec{c}.\vec{b}+\vec{c}.\vec{c} = 0 \quad \mathbf{1}$$

$$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$$

$$\Rightarrow 3^2+5^2+7^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0 \quad ½$$

$$\Rightarrow 2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = -(9+25+49)$$

$$\Rightarrow (\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = -\frac{83}{2} \quad ½$$

[CBSE SQP Marking Scheme, 2020]

Q. 4. If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

R&U **[CBSE Delhi Set-III, 2019]**

Sol. Given $|\hat{a}+\hat{b}| = 1$

as $|\hat{a}+\hat{b}|^2+|\hat{a}-\hat{b}|^2 = 2(|\hat{a}|^2+|\hat{b}|^2)$ **1**

$\Rightarrow 1+|\vec{a}-\vec{b}|^2 = 2(1+1)$

$\Rightarrow |\vec{a}-\vec{b}|^2 = 3 \Rightarrow |\vec{a}-\vec{b}| = \sqrt{3}$ **1**

[CBSE Marking Scheme, 2019]

Detailed Solution :

Let the two unit vectors be $\vec{a}$ and $\vec{b}$

Given, $|\vec{a}+\vec{b}| = 1$

$$\Rightarrow \quad |\vec{a}+\vec{b}|^2 = 1$$

$$\Rightarrow \quad |\vec{a}|^2+|\vec{b}|^2+2\vec{a}.\vec{b} = 1$$

$$\Rightarrow \quad 1+1+2\vec{a}.\vec{b} = 1 \qquad (\because |\vec{a}| = 1 = |\vec{b}|)$$

$$\Rightarrow \quad 2\vec{a}.\vec{b} = -1 \qquad ...(i)$$

Also, $|\vec{a}-\vec{b}|^2 = |\vec{a}|^2+|\vec{b}|^2-2\vec{a}.\vec{b}$

$= 1 + 1 - (-1)$ [from eq. (i)]

$= 2 + 1 = 3$

$\therefore \quad |\vec{a}-\vec{b}| = \sqrt{3}$ **Hence Proved.**

Commonly Made Error

- Mostly students fail to understand the question.

Answering Tip

- Practice all types of questions from dot products.

Q. 5. Find the projection (vector) of $2\hat{i}-\hat{j}+\hat{k}$ on $\hat{i}-2\hat{j}+\hat{k}$.

R&U **[SQP 2017-18]**

Short Answer Type Questions-II (3 marks each)

Q. 1. $\vec{r}$ and $\vec{s}$ are unit vectors. If $|\vec{r}+\vec{s}| = \sqrt{2}$, find:

(i) the angle between $\vec{r}$ and $\vec{s}$.

(ii) the value of $(4\vec{r}-\vec{s}).(2\vec{r}+\vec{s})$.

Show your steps.

A I R&U **[CBSE Practice Questions 2022]**

Sol. (i) Given, $|\vec{r}| = 1$, $|\vec{s}| = 1$

and $|\vec{r}+\vec{s}| = \sqrt{2}$

Now, $|\vec{r}+\vec{s}|^2 = (\sqrt{2})^2$

$$\Rightarrow |\vec{r}|^2+|\vec{s}|^2+2\vec{r}.\vec{s} = 2$$

$$\Rightarrow 1^2+1^2+2\vec{r}.\vec{s} = 2$$

$$\Rightarrow 2\vec{r}.\vec{s} = 0$$

$$\Rightarrow \vec{r}.\vec{s} = 0$$

$$\Rightarrow |\vec{r}||\vec{s}|\cos\theta = 0$$

$$\Rightarrow (1).(1).\cos\theta = 0$$

$$\Rightarrow \cos\theta = \cos\frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

(ii) Given, $|\vec{r}| = 1$, $|\vec{s}| = 1$

$$(4\vec{r}-\vec{s}).(2\vec{r}+\vec{s}) = 8|\vec{r}|^2 + 4\vec{r}.\vec{s} - 2\vec{s}.\vec{r} - |\vec{s}|^2$$

$= 8(1)+4|\vec{r}||\vec{s}|\cos\theta - 2|\vec{s}||\vec{r}|\cos\theta - (1)^2$

$= 8 + 4(1)(1)\cos\theta - 2(1)(1)\cos\theta - 1$

$= 8 + 0 - 0 - 1$

$= 7$ [$\because \cos\theta = 0$ from above part]

Q. 2. If $\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b} = 2\hat{i}+4\hat{j}-5\hat{k}$ represent two adjacent sides of a parallelogram, find unit vectors parallel to the diagonals of the parallelogram. R&U [CBSE Delhi Set-I 2020]

Sol. Diagonal vectors are :

$$\vec{a}+\vec{b} = 3\hat{i}+6\hat{j}-2\hat{k}$$

and $$\vec{a}-\vec{b} = -\hat{i}-2\hat{j}+8\hat{k}$$

(or, $\vec{b}-\vec{a} = \hat{i}+2\hat{j}-8\hat{k}$) **1**

$\therefore$ unit vectors parallel to the diagonals are

$$\left(\frac{(\vec{a}+\vec{b})}{|\vec{a}+\vec{b}|}\right) = \frac{3}{7}\hat{i}+\frac{6}{7}\hat{j}-\frac{2}{7}\hat{k}$$

and $$\left(\frac{(\vec{a}-\vec{b})}{|\vec{a}-\vec{b}|}\right) = \frac{1}{\sqrt{69}}\hat{i}+\frac{2}{\sqrt{69}}\hat{j}-\frac{8}{\sqrt{69}}\hat{k}$$ **1+1**

[CBSE Marking Scheme 2020 (Modified)]

Detailed Solution:

Given that, $\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b} = 2\hat{i}+4\hat{j}-5\hat{k}$ are two adjacent sides of a parallelogram.

Let us suppose $\vec{d_1}$ and $\vec{d_2}$ are two diagonals of parallelogram.

Then,
$$\vec{d_1} = \vec{a}+\vec{b}$$
$$= \hat{i}+2\hat{j}+3\hat{k}+2\hat{i}+4\hat{j}-5\hat{k}$$
$$= 3\hat{i}+6\hat{j}-2\hat{k}$$

and
$$\vec{d_2} = \vec{b}-\vec{a}$$
$$= 2\hat{i}+4\hat{j}-5\hat{k}-\hat{i}-2\hat{j}-3\hat{k}$$
$$= \hat{i}+2\hat{j}-8\hat{k}$$

Now, unit vector parallel to $\vec{d_1}$ is

$$\hat{d_1} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{9+36+4}} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{49}}$$

$$\hat{d_1} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{7}$$

And unit vector parallel to $\vec{d_2}$ is

$$\vec{d_2} = \frac{\hat{i}+2\hat{j}-8\hat{k}}{\sqrt{1+4+64}}$$

$$= \frac{\hat{i}+2\hat{j}-8\hat{k}}{\sqrt{69}}$$

Commonly Made Error

- Instead of finding the parallel vectors, some students take the cross product to find the perpendicular vector.

Answering Tip

- Practice problems based on parallel and perpendicular vectors.

Q. 3. If $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+5\hat{j}$, $3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ respectively are the position vectors of points *A*, *B*, *C* and *D*, then find the angle between the straight lines *AB* and *CD*. Find whether $\vec{AB}$ and $\vec{CD}$ are collinear or not. R&U [CBSE Delhi Set-III, 2019]

Q. 4. The scalar product of the vector $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $\vec{b} = 2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c} = \lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b}+\vec{c}$. A I R&U [CBSE OD Set-I, 2019]

Sol. Given that

$$\vec{a}\cdot\frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|} = 1$$ ½

or $$\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c} = |\vec{b}+\vec{c}|$$ ½

$$(\hat{i}+\hat{j}+\hat{k})\cdot(2\hat{i}+4\hat{j}-5\hat{k})+(\hat{i}+\hat{j}+\hat{k})\cdot(\lambda\hat{i}+2\hat{j}+3\hat{k})$$

$$= |(\lambda+2)\hat{i}+6\hat{j}-2\hat{k}|$$ ½

or $(2 + 4 - 5) + (\lambda + 2 + 3)$

$$= \left|\sqrt{(\lambda+2)^2+36+4}\right|$$ ½

$\therefore$ $(\lambda + 6)^2 = (\lambda + 2)^2 + 40$ or $\lambda = 1$ ½

Hence $$\frac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{7}$$

$$= \frac{3}{7}\hat{i}+\frac{6}{7}\hat{j}-\frac{2}{7}\hat{k}$$ ½

[CBSE Marking Scheme 2019 (Modified)]

Q. 5. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$. Also, find the angle which $\vec{a}+\vec{b}+\vec{c}$ makes with $\vec{a}$ or $\vec{b}$ or $\vec{c}$.

A [Delhi 2017]

Sol. $|\vec{a}| = |\vec{b}| = |\vec{c}|$ and $\vec{a}.\vec{b} = 0 = \vec{b}.\vec{c} = \vec{c}.\vec{a}$ ...(i) ½

Let α, β and γ be the angles made by $(\vec{a}+\vec{b}+\vec{c})$ with $\vec{a}, \vec{b}$ and $\vec{c}$ respectively

$$(\vec{a}+\vec{b}+\vec{c}).\vec{a} = |\vec{a}+\vec{b}+\vec{c}||\vec{a}| \cos\alpha$$

or $$\alpha = \cos^{-1}\left(\frac{|\vec{a}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$$

Similarly, $\beta = \cos^{-1}\left(\frac{|\vec{b}|}{|\vec{a}+\vec{b}+\vec{c}|}\right)$ and

$$\gamma = \cos^{-1}\left(\frac{|\vec{c}|}{|\vec{a}+\vec{b}+\vec{c}|}\right) \quad \mathbf{1}$$

using (i), we get α = β = γ

Now

$$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})$$

1

or $$|\vec{a}+\vec{b}+\vec{c}|^2 = 3|\vec{a}|^2 \text{ (using (i))}$$

or $$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}|\vec{a}|$$

∴ $$= \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = \beta = \gamma \quad ½$$

[CBSE Marking Scheme, 2017 (Modified)]

Q. 6. If $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, find the angles which the vector $2\vec{a}+\vec{b}+2\vec{c}$ makes with the vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

A [O.D. Comptt. 2017]

Sol. Let the vector $\vec{P} = (2\vec{a}+\vec{b}+2\vec{c})$ makes angles α, β, γ respectively with the vector $\vec{a}, \vec{b}, \vec{c}$.

Given that $|\vec{a}| = |\vec{b}| = |\vec{c}|$ and $\vec{a}.\vec{b} = \vec{c}.\vec{a}$

$$= \vec{b}.\vec{c} = 0 \quad ½$$

$$|\vec{P}|^2 = (2\vec{a}+\vec{b}+2\vec{c}).(2\vec{a}+\vec{b}+2\vec{c})$$

$$|\vec{P}|^2 = 4\vec{a}.\vec{a}+2\vec{a}\vec{b}+4\vec{a}.\vec{c}+2\vec{b}.\vec{a}+\vec{b}.\vec{b}+2\vec{b}.\vec{c}+4\vec{c}.\vec{a}+2\vec{c}.\vec{b}+4\vec{c}.\vec{c}$$

$$|\vec{P}|^2 = 4|\vec{a}|^2+|\vec{b}|^2+4|\vec{c}|^2$$

$$|\vec{P}|^2 = 9|\vec{a}|^2$$

$$|\vec{P}| = 3|\vec{a}|$$

$$\cos\alpha = \frac{(2\vec{a}+\vec{b}+2\vec{c}).\vec{a}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{a}|} \quad ½$$

$$= \frac{2|\vec{a}|^2}{3|\vec{a}||\vec{a}|}$$

$$= \frac{2}{3} \text{ or } \alpha = \cos^{-1}\frac{2}{3} \quad \mathbf{1}$$

$$\cos\beta = \frac{(2\vec{a}+\vec{b}+2\vec{c}).\vec{b}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{b}|}$$

$$= \frac{|\vec{b}|^2}{3|\vec{b}||\vec{b}|}$$

$$= \frac{1}{3}$$

or $$\beta = \cos^{-1}\frac{1}{3} \quad ½$$

$$\cos\gamma = \frac{(2\vec{a}+\vec{b}+2\vec{c}).\vec{c}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{c}|}$$

$$= \frac{2|\vec{c}|^2}{3|\vec{c}||\vec{c}|}$$

$$= \frac{2}{3}$$

or $$\gamma = \cos^{-1}\frac{2}{3} \quad ½$$

[CBSE Marking Scheme, 2017 (Modified)]

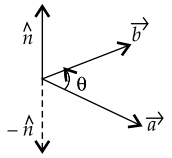
Topic-3 Cross Product

Concepts Covered • *Properties of cross product,* • *Relationship between Vector product and scalar product*

Revision Notes

1. The cross product or vector product of two vectors $\vec{a}$ and $\vec{b}$ is defined by,

$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \, \hat{n}$, where θ is the angle between the vectors $\vec{a}$ and $\vec{b}$, $0 \le \theta \le \pi$ and $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. For better illustration, see figure.



Consider $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$.

then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} =$$

$$(a_2b_3 - a_3b_2)\hat{i} - (a_1b_3 - a_3b_1)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}.$$

- **Properties/Observations of Cross Product**

➲ $\hat{i} \times \hat{i} = |\hat{i}||\hat{i}| \sin 0 = \vec{0}$ or $\hat{i} \times \hat{i} = 0 = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}$.

➲ $\hat{i} \times \hat{j} = |\hat{i}||\hat{j}| \sin\frac{\pi}{2} . \hat{k} = \hat{k}$ or $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$.

➲ $\vec{a} \times \vec{b}$ is a vector $\vec{c}$ (say) then this vector $\vec{c}$ is perpendicular to both the vectors $\vec{a}$ and $\vec{b}$.

➲ $\vec{a} \times \vec{b} = \vec{0} \Leftrightarrow \vec{a} \,||\, \vec{b}$ or, $\vec{a} = \vec{0}$, $\vec{b} = \vec{0}$.

➲ $\vec{a} \times \vec{a} = \vec{0}$.

➲ $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ (Commutative property does not hold for cross product).

➲ $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$ (Left distributive).

➲ $(\vec{b} + \vec{c}) \times \vec{a} = \vec{b} \times \vec{a} + \vec{c} \times \vec{a}$ (Right distributive).

(Distributive property of the vector product or cross product)

2. Relationship between Vector product and Scalar product [Lagrange's Identity]

or $|\vec{a} \times \vec{b}|^2 + \left(\vec{a} \cdot \vec{b}\right)^2 = |\vec{a}|^2 \cdot |\vec{b}|$

3. Cauchy-Schwarz inequality :

For any two vectors $\vec{a}$ and $\vec{b}$,

always have $|\vec{a} \cdot \vec{b}| \le |\vec{a}||\vec{b}|$.

Note :

- If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle, then the area of triangle can be obtained by evaluating $\frac{1}{2}|\vec{a} \times \vec{b}|$.
- If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a parallelogram, then the area of parallelogram can be obtained by evaluating $|\vec{a} \times \vec{b}|$.
- The area of the parallelogram with diagonals $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}|\vec{a} \times \vec{b}|$.

Key Formulae

➲ Angle between two vectors $\vec{a}$ and $\vec{b}$ in terms of cross-product can be found by the expression given here :

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} \text{ or } \theta = \sin^{-1}\left(\frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}\right).$$

OBJECTIVE TYPE QUESTIONS

A Multiple Choice Questions

Q. 1. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to

(A) 0 (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ (D) π

Ans. Option (B) is correct.

Explanation: Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors, so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

$$|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$$

$$\Rightarrow \quad |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$$

$$\Rightarrow \quad \cos\theta = \sin\theta$$

$$\left[\because |\vec{a}| \text{ and } |\vec{b}| \text{ are positive.}\right]$$

$$\Rightarrow \quad \tan\theta = 1$$

$$\Rightarrow \quad \theta = \frac{\pi}{4}$$

So that, $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to $\frac{\pi}{4}$.

Q. 2. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Ans. Option (B) is correct.

Explanation:

It is given that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$.

We know that $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\, \hat{n}$, where $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ and θ is the angle between $\vec{a}$ and $\vec{b}$.

Now, $\vec{a} \times \vec{b}$ is a unit vector if

$$|\vec{a} \times \vec{b}| = 1$$

$$\Rightarrow \quad \left||\vec{a}||\vec{b}|\sin\theta\hat{n}\right| = 1$$

$$\Rightarrow \quad |\vec{a}||\vec{b}||\sin\theta| = 1$$

$$\Rightarrow \quad 3 \times \frac{\sqrt{2}}{3} \times \sin\theta = 1$$

$$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \theta = \frac{\pi}{4}$$

So that, $\vec{a} \times \vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$.

Q. 3. Area of a rectangle having vertices *A, B, C* and *D* with position vectors $-\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$, $\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$, $\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$ and $-\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$, respectively is

(A) $\frac{1}{2}$ unit^2 (B) 1 unit^2

(C) 2 unit^2 (D) 4 unit^2

Ans. Option (C) is correct.

Explanation: The position vectors of vertices *A, B, C* and *D* of rectangle *ABCD* are given as:

$$\overrightarrow{OA} = -\hat{i} + \frac{1}{2}\hat{j} + 4\hat{k}$$

$$\overrightarrow{OB} = \hat{i} + \frac{1}{2}\hat{j} + 4\hat{k},$$

$$\overrightarrow{OC} = \hat{i} - \frac{1}{2}\hat{j} + 4\hat{k},$$

$$\overrightarrow{OD} = -\hat{i} - \frac{1}{2}\hat{j} + 4\hat{k}$$

The adjacent sides $\overrightarrow{AB}$ and $\overrightarrow{BC}$ of the given rectangle are given as :

$$\overrightarrow{AB} = (1+1)\hat{i} + \left(\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4-4)\hat{k}$$

$$= 2\hat{i}$$

$$\overrightarrow{BC} = (1-1)\hat{i} + \left(-\frac{1}{2} - \frac{1}{2}\right)\hat{j} + (4-4)\hat{k}$$

$$= -\hat{j}$$

$$\therefore \quad \overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}$$

$$= \hat{k}(-2)$$

$$= -2\hat{k}$$

$$\Rightarrow \quad |\overrightarrow{AB} \times \overrightarrow{BC}| = 2 \text{ unit}^2$$

Now, it is known that the area of parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \vec{b}|$.

Therefore, the area of the given rectangle is $|\overrightarrow{AB} \times \overrightarrow{BC}| = 2$sq. units.

Q. 5. The vectors from origin to the points *A* and *B* are $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$ respectively, then the area of triangle *OAB* is

(A) 340 (B) $\sqrt{25}$

(C) $\sqrt{229}$ (D) $\frac{1}{2}\sqrt{229}$

Ans. Option (D) is correct.

Explanation:

$$\text{Area of } \Delta OAB = \frac{1}{2}\left|\overline{OA} \times \overline{OB}\right|$$
$$= \frac{1}{2}\left|(2\hat{i} - 3\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} + \hat{k})\right|$$
$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$
$$= \frac{1}{2}\left|\left[\hat{i}(-3-6) - \hat{j}(2-4) + \hat{k}(6+6)\right]\right|$$
$$= \frac{1}{2}\left|-9i + 2\hat{j} + 12\hat{k}\right|$$
$$\text{Area of } \Delta OAB = \frac{1}{2}\left|\sqrt{(81+4+144)}\right|$$
$$= \frac{1}{2}\sqrt{229} \text{ square units}$$

Q. 6. For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is equal to

(A) $\vec{a}^{\,2}$ **(B)** $3\vec{a}^{\,2}$

(C) $4\vec{a}^{\,2}$ **(D)** $2\vec{a}^{\,2}$

Ans. Option (D) is correct.

Explanation:

Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore \quad \vec{a}^2 = x^2 + y^2 + z^2$

$$\therefore \quad \vec{a} \times \hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 0 & 0 \end{vmatrix}$$
$$= \hat{i}[0] - \hat{j}[-z] + \hat{k}[-y]$$
$$= z\hat{j} - y\hat{k}$$
$$\therefore \quad (\vec{a} \times \hat{i})^2 = (z\hat{j} - y\hat{k})(z\hat{j} - y\hat{k})$$
$$= y^2 + z^2$$

Similarly, $(\vec{a} \times \hat{j})^2 = x^2 + z^2$ and $(\vec{a} \times \hat{k})^2 = x^2 + y^2$

$$(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2 = y^2 + z^2 + x^2 + z^2 + x^2 + y^2$$
$$= 2(x^2 + y^2 + z^2)$$
$$= 2\vec{a}^2$$

Q. 7. If $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then the value of $|\vec{a} \times \vec{b}|$ is

(A) 5 **(B)** 10

(C) 14 **(D)** 16

Ans. Option (D) is correct.

Explanation:

Here, $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$ [Given]

$$\therefore \quad \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$
$$12 = 10 \times 2\cos\theta$$
$$\Rightarrow \quad \cos\theta = \frac{12}{20}$$
$$= \frac{3}{5}$$
$$\Rightarrow \quad \sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{9}{25}}$$
$$\sin\theta = \pm\frac{4}{5}$$
$$\therefore \quad |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}||\sin\theta|$$
$$= 10 \times 2 \times \frac{4}{5}$$
$$= 16$$

Q. 8. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is

(A) one **(B)** two

(C) three **(D)** infinite

Ans. Option (B) is correct.

Explanation: The number of vectors of unit length perpendicular to the vectors $\vec{a}$ and $\vec{b}$ is c (say)

i.e., $\quad \vec{c} = \pm(\vec{a} \times \vec{b})$.

So, there will be two vectors of unit length perpendicular to the vectors $\vec{a}$ and $\vec{b}$.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (1 mark each)

Q. 1. Find the area of the triangle whose two sides are represented by the vectors $2\hat{i}$ and $-3\hat{j}$.

R [CBSE SQP 2020]

Q. 2. Write the angle between the vectors $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$. [Delhi Comptt. 2017]

Q. 3. If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 225$ and $|\vec{a}| = 5$, then write the value of $|\vec{b}|$. A I R&U [O.D. Comptt. 2017]

Sol.
$$(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 225$$
$$\text{or} \quad |\vec{a}|^2|\vec{b}|^2(\sin^2\theta + \cos^2\theta) = 225$$
$$\text{or} \quad (5)^2|\vec{b}|^2 = 225 \text{ or } |\vec{b}| = 3 \quad \mathbf{1}$$

[CBSE Marking Scheme 2017]

Q. 4. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ having the same length $\sqrt{2}$ and their vector product is $-\hat{i}-\hat{j}+\hat{k}$.

R&U [Outside Delhi Set I, II, III Comptt. 2016]

Sol.
$$\sin\theta = \frac{|-\hat{i}-\hat{j}+\hat{k}|}{\sqrt{2}\cdot\sqrt{2}}$$
$$\sin\theta = \frac{\sqrt{3}}{2}$$
or $\theta = 60°$

or $\theta = \dfrac{\pi}{3}$ **1**

[CBSE Marking Scheme 2016]

Q. 5. Find λ and μ if $(\hat{i}+3\hat{j}+9\hat{k})\times(3\hat{i}-\lambda\hat{j}+\mu\hat{k}) = 0$.

A I R&U [O.D. 2016]

Sol. Getting $\lambda = -9$

and $\mu = 27$ **1**

[CBSE Marking Scheme 2016]

Detailed Solution :

$$(\hat{i}+3\hat{j}+9\hat{k})\times(3\hat{i}-\lambda\hat{j}+\mu\hat{k}) = 0$$

or
$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 9 \\ 3 & -\lambda & \mu \end{vmatrix} = 0$$

or $\hat{i}(3\mu+9\lambda)-\hat{j}(\mu-27)+\hat{k}(-\lambda-9) = 0$

or $3\mu + 9\lambda = 0$ ...(i)

or $\mu - 27 = 0$ ...(ii)

or $-\lambda - 9 = 0$ ...(iii)

from eqn. (ii) and (iii),

$\mu = 27$

and $\lambda = -9$

Q. 6. If vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \dfrac{1}{2}, |\vec{b}| = \dfrac{4}{\sqrt{3}}$ and $|\vec{a}\times\vec{b}| = \dfrac{1}{\sqrt{3}}$, then find $|\vec{a}\cdot\vec{b}|$.

R&U [O.D. Set II 2016]

Topper Answer, 2016

Sol. $|\bar{a}| = \frac{1}{2}$ $|\bar{b}| = \frac{4}{\sqrt{3}}$ $|\bar{a}\times\bar{b}| = \frac{1}{\sqrt{3}}$ $|\bar{a}\cdot\bar{b}| = ?$

$|\bar{a}\times\bar{b}| = |\bar{a}||\bar{b}|\sin\theta$ (θ: Angle between vectors $\bar{a}$ & $\bar{b}$)

$\frac{1}{\sqrt{3}} = \frac{1}{2}\times\frac{4}{\sqrt{3}}\sin\theta$

$\sin\theta = \frac{1}{2}$ $\theta = 30°$

$|\bar{a}\cdot\bar{b}| = |\bar{a}||\bar{b}|\cos\theta$

$= \frac{1}{2}\times\frac{4}{\sqrt{3}}\times\frac{\sqrt{3}}{2} = 1$

$\therefore |\bar{a}\cdot\bar{b}| = 1$

Short Answer Type Questions-I (2 marks each)

Q. 1. Find a unit vector perpendicular to each of the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = 5\hat{i}+6\hat{j}-2\hat{k}$ and $\vec{b} = 7\hat{i}+6\hat{j}+2\hat{k}$.

[CBSE Delhi Set-II, 2020]

Topper Answer, 2020

Sol. $\vec{a} = 5\hat{i} + 6\hat{j} - 2\hat{k}$

$$\begin{vmatrix} 5 & 6 & -2 \\ 7 & 6 & 2 \end{vmatrix}$$

$\therefore \quad \vec{c} = \pm \lambda (24\hat{i} - 24\hat{j} - 12\hat{k})$

$= \pm \beta (2\hat{i} - 2\hat{j} - \hat{k}) \quad , \quad \beta = 12\lambda$

$\hat{c} = \pm \frac{1}{3} (2\hat{i} - 2\hat{j} - \hat{k}) \quad$ as $\sqrt{2^2+2^2+1^2} = 3$

$\therefore \quad \hat{c} = \pm \left(\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k} \right)$

Q. 2. Find the area of the parallelogram whose one side and a diagonal are represented by coinitial vectors $\hat{i} - \hat{j} + \hat{k}$ and $4\hat{i} + 5\hat{k}$ respectively.

A I R&U [CBSE SQP 2020-21]

Sol. Let

$$\vec{a} = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{d} = 4\hat{i} + 5\hat{k}$$

$$\because \quad \vec{a} + \vec{b} = \vec{d}$$

$$\therefore \quad \vec{b} = \vec{d} - \vec{a}$$

$$= 3\hat{i} + \hat{j} + 4\hat{k} \qquad ½$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 3 & 1 & 4 \end{vmatrix}$$

$$= -5\hat{i} - 1\hat{j} + 4\hat{k} \qquad 1$$

Area of parallelogram $= |\vec{a} \times \vec{b}|$

$$= \left|\sqrt{25 + 1 + 16}\right|$$

$$= \sqrt{42} \text{ sq. units} \qquad ½$$

[CBSE SQP Marking Scheme 2020-21]

Commonly Made Error

- Mostly students get confused in deciding the formula to be used since a side and a diagonal are given.

Answering Tip

- Practice more problems related to area of triangle and parallelogram.

Q. 3. Find the unit vector perpendicular to each of the vectors $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$.

R&U [CBSE Delhi Set-I, II, III 2020]

Q. 4. If $|\vec{a}| = 2, |\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

R&U [CBSE OD Set-I, 2019]

Q. 5. If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$, find sin θ.

R&U [Delhi/O.D.-2018]

Sol.

$$\sin\theta = \frac{|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})|}{|\hat{i} - 2\hat{j} + 3\hat{j}|\,|3\hat{i} - 2\hat{j} + \hat{k}|} \qquad ½$$

$$|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})|$$

$$= |4\hat{i} + 8\hat{j} + 4\hat{k}| = 4\sqrt{6} \qquad 1$$

$$\sin\theta = \frac{4\sqrt{6}}{14} = \frac{2\sqrt{6}}{7} \qquad ½$$

[CBSE Marking Scheme, 2018]

Detailed Solution :

Topper Answer, 2018

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

$|\vec{a}| = \sqrt{1+4+9} = \sqrt{14}$

$\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{b}| = \sqrt{9+4+1} = \sqrt{14}$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 3 & -2 & 1 \end{vmatrix}$

$= 4\hat{i} + 8\hat{j} + 4\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{16+64+16} = \sqrt{96} = 4\sqrt{6}$

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

$4\sqrt{6} = \sqrt{14} \times \sqrt{14} \times \sin\theta$

$4\sqrt{6} = 14 \sin\theta$

$\sin\theta = \frac{4\sqrt{6}}{14} = \frac{2\sqrt{6}}{7}$

$\theta = \sin^{-1}\left(\frac{2\sqrt{6}}{7}\right)$ Ans.

Q. 6. Using vectors, find the area of triangle ABC, with vertices $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$.

A I R&U **[Foreign 2017]**

Sol. $\vec{AB} = \hat{i} - 3\hat{j} + \hat{k}$, $\vec{AC} = 3\hat{i} + 3\hat{j} - 4\hat{k}$ ½+½

$$\text{Area of } \Delta ABC = \frac{1}{2}|\vec{AB} \times \vec{AC}|$$

$$= \frac{1}{2} \text{ magnitude of } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix} \qquad 1$$

$$= \frac{1}{2}(9\hat{i} + 7\hat{j} + 12\hat{k}) = \frac{\sqrt{274}}{2} \text{ sq. units}$$

[CBSE Marking Scheme 2017]

Short Answer Type Questions-II (3 marks each)

Q. 1. Three vertices A, B, C and D of a parallelogram ABCD are given by, A(0, –3, 3), B(–5, m – 3, 0) and D(1, –3, 4). The area of the parallelogram ABCD is 6 sq. units. Using vector method, find the value(s) of m. Show your steps.

R **[CBSE Practice Questions 2022]**

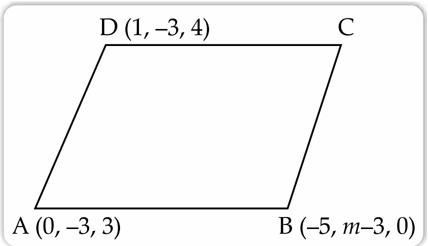
Sol. Area of ||gm ABCD = $|\overrightarrow{AB} \times \overrightarrow{AD}|$

Now, $\overrightarrow{AB} = (-5-0)\hat{i} + (m-3-(-3))\hat{j} + (0-3)\hat{k}$

or, $\overrightarrow{AB} = -5\hat{i} + m\hat{j} - 3\hat{k}$ ½

and $\overrightarrow{AD} = (1-0)\hat{i} + (-3-(-3))\hat{j} + (4-3)\hat{k}$

or, $\overrightarrow{AD} = \hat{i} + \hat{k}$ ½



$$\therefore \quad \overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & m & -3 \\ 1 & 0 & 1 \end{vmatrix}$$

or, $\overrightarrow{AB} \times \overrightarrow{AD} = \hat{i}(-m-0) - \hat{j}(-5+3) + \hat{k}(0-m)$

or, $\overrightarrow{AB} \times \overrightarrow{AD} = -m\hat{i} + 2\hat{j} - m\hat{k}$ ½

Now, $|\overrightarrow{AB} \times \overrightarrow{AD}| = \left|\sqrt{(-m)^2 + (2)^2 + (-m)^2}\right|$

or, $|\overrightarrow{AB} \times \overrightarrow{AD}| = \left|\sqrt{m^2 + 4 + m^2}\right|$

$= \left|\sqrt{2m^2 + 4}\right|$ ½

Given, area of parallelogram ABCD

= 6 sq. units

$\therefore \quad \left|\sqrt{2m^2 + 4}\right| = 6$

$\Rightarrow \quad 2m^2 + 4 = 36$

$\Rightarrow \quad 2m^2 = 32$

$\Rightarrow \quad m^2 = 16$

$\Rightarrow \quad m = \pm 4$

Thus, value of m is ±4. 1

Q. 2. If $\vec{a} \neq \vec{0}$, $\vec{a}.\vec{b} = \vec{a}.\vec{c}$, $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then show that $\vec{b} = \vec{c}$

R&U **[SQP 2021-22]**

Sol. We have $\vec{a}.(\vec{b}-\vec{c}) = 0$

$$\Rightarrow \quad (\vec{b}-\vec{c}) = \vec{0} \text{ or } \vec{a} \perp r(\vec{b}-\vec{c})$$

$$\Rightarrow \quad \vec{b} = \vec{c} \text{ or } \vec{a} \perp r(\vec{b}-\vec{c}) \qquad \mathbf{1}$$

Also, $\vec{a} \times (\vec{b}-\vec{c}) = \vec{0}$

$$\Rightarrow \quad (\vec{b}-\vec{c}) = \vec{0} \text{ or } \vec{a} \,||\, (\vec{b}-\vec{c})$$

$$\Rightarrow \quad \vec{b} = \vec{c} \text{ or } \vec{a} \,||\, (\vec{b}-\vec{c}) \qquad \mathbf{1}$$

$\vec{a}$ cannot be both perpendicular to $(\vec{b}-\vec{c})$ and parallel to $(\vec{b}-\vec{c})$.

Hence, $\vec{b} = \vec{c}$

[CBSE Marking Scheme 2022]

Q. 3. Using vectors, find the area of the triangle whose vertices are $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$. R&U [Delhi 2017]

[CBSE Delhi Set I, II, III-2020]

Sol. Given, $\vec{OA} = \hat{i}+2\hat{j}+3\hat{k}$

$$\vec{OB} = 2\hat{i}-\hat{j}+4\hat{k}$$

and $\vec{OC} = 4\hat{i}+5\hat{j}-\hat{k}$

Now, $\vec{AB} = \vec{OB}-\vec{OA}$

$$= (2\hat{i}-\hat{j}+4\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})$$

$$= \hat{i}-3\hat{j}+\hat{k} \qquad \tfrac{1}{2}$$

and $\vec{AC} = \vec{OC}-\vec{OA}$

$$= (4\hat{i}+5\hat{j}-\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})$$

$$= 3\hat{i}+3\hat{j}-4\hat{k} \qquad \tfrac{1}{2}$$

$\therefore$ The area of the given triangle

$$= \frac{1}{2}|\vec{AB}\times\vec{AC}|$$

Now, $$\vec{AB}\times\vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix}$$

$$= \hat{i}\,(12-3) + \hat{j}\,(3+4) + \hat{k}\,(3+9)$$

$$= 9\hat{i} + 7\hat{j} + 12\hat{k} \qquad \mathbf{1}$$

Therefore,

$$|\vec{AB}\times\vec{AC}| = \left|\sqrt{(9)^2+(7)^2+(12)^2}\right|$$

$$= \left|\sqrt{81+49+144}\right|$$

$$= \sqrt{274} \text{ unit}^2$$

Hence,

$$\text{required area} = \frac{1}{2}\sqrt{274}. \text{ unit}^2 \qquad \mathbf{1}$$

[CBSE Marking Scheme 2020 (Modified)]

Q. 4. Let $\vec{a} = 4\hat{i}+5\hat{j}-\hat{k}$, $\vec{b} = \hat{i}-4\hat{j}+5\hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}-\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{c}$ and $\vec{b}$ and satisfying $\vec{d}\cdot\vec{c} = 21$. A I R&U [CBSE OD/Delhi 2018]

Sol. $$\vec{d} = \lambda(\vec{c}\times\vec{b}) = \lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 1 & -4 & 5 \end{vmatrix} \qquad \mathbf{1}$$

$$\therefore \quad \vec{d} = \lambda\hat{i}-16\lambda\hat{j}-13\lambda\hat{k}$$

$$\vec{d}\cdot\vec{a} = 21 \Rightarrow 4\lambda - 80\lambda + 13\lambda = 21$$

$$\lambda = -\frac{1}{3} \qquad \mathbf{1}$$

$$\therefore \quad \vec{d} = -\frac{1}{3}\hat{i}+\frac{16}{3}\hat{j}+\frac{13}{3}\hat{k} \qquad \mathbf{1}$$

[CBSE Marking Scheme 2018 (Modified)]

OR

Topper Answer, 2018

Sol. let $\vec{d}$ be $x\hat{i}+y\hat{j}+z\hat{k}$

Now, $\vec{d} \perp \vec{c}$

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(3\hat{i}+\hat{j}-\hat{k}) = 0$

$3x+y-z=0$ —— (i)

Also, $\vec{d} \perp \vec{b}$

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}-4\hat{j}+5\hat{k}) = 0$

$x-4y+5z=0$ —— (ii)

Also, $\vec{d}\cdot\vec{a} = 21$

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(4\hat{i}+5\hat{j}-\hat{k}) = 21$

$4x+5y-z=21$ —— (iii)

Solving (i) and (iii) equation

$3x+y-\cancel{z}=0$

$-4x+5y\cancel{-z}=21$

$-x-4y=-21$

$\Rightarrow x+4y=21$ —— (iv)

Multiply (i) by 5 and add to (ii) eq.

$15x+05y-5\cancel{z}=0$

$+x-4y+\cancel{5z}=0$

$16x+y=0$ —— (v)

Solving (iv) and (v) eq.

$x+4y=21$

$-64x+4y=0$

63 21 3

Putting value of x in eq. (v).

$x + 4y = 21$

$\frac{-1}{3} + 4y = 21$

$4y = 21 + \frac{1}{3}$

$4y = \frac{63+1}{3}$

$4y = \frac{64}{3}$

$y = \frac{64}{12} = \frac{32}{6} = \frac{16}{3}$

Also, $3x + y - z = 0$

$3\left(\frac{-1}{3}\right) + \frac{16}{3} = z$

$-1 + \frac{16}{3} = z$

$\frac{-3+16}{3} = z$, $z = \frac{13}{3}$

Therefore, $\boxed{\vec{x} = -\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}}$ Ans

$= \frac{1}{3}(-\hat{i} + 16\hat{j} + 13\hat{k})$ Ans.

Q. 5. If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = 4\hat{i} - 7\hat{j} + \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 6$.

K&U [Foreign 2017]

Sol. Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$; $\vec{a} \cdot \vec{c} = 6$ or $2x + y - z = 6$

Now, $\vec{a} \times \vec{c} = \vec{b}$

or $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ x & y & z \end{vmatrix} = 4\hat{i} - 7\hat{j} + \hat{k}$ **1**

Sol.

or $\hat{i}(z+y) - \hat{j}(2z+x) + \hat{k}(2y-x) = 4\hat{i} - 7\hat{j} + \hat{k}$

or $z + y = 4, 2z + x = 7, 2y - x = 1$ **1**

Solving and getting $x = 3, y = 2, z = 2$

$\vec{c} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ **1**

[CBSE Marking Scheme 2017 (modified)]

Q. 6. Find the area of a parallelogram *ABCD* whose side *AB* and the diagonal *AC* are given by the vectors $3\hat{i} + \hat{j} + 4\hat{k}$ and $4\hat{i} + 5\hat{k}$ respectively.

R&U [Foreign 2017]

OR

Q. 7. If $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 2\hat{j} - 3\hat{k}$ then express $\vec{b}$ in the form of $\vec{b} = \vec{b_1} + \vec{b_2}$, where $\vec{b_1}$ is parallel to $\vec{a}$ and $\vec{b_2}$ is perpendicular to $\vec{a}$.

AI R&U [O.D. 2017]

Sol. $\vec{b_1} \| \vec{a}$ or let $\vec{b_1} = \lambda(2\hat{i} - \hat{j} - 2\hat{k})$ ½

$\vec{b_2} = \vec{b} - \vec{b_1}$

$= (7\hat{i} + 2\hat{j} - 3\hat{k}) - (2\lambda\hat{i} - \lambda\hat{j} - 2\lambda\hat{k})$ ½

$= (7 - 2\lambda)\hat{i} + (2 + \lambda)\hat{j} - (3 - 2\lambda)\hat{k}$ ½

$\vec{b_2} \perp \vec{a}$ or $2(7 - 2\lambda) - 1(2 + \lambda) + 2(3 - 2\lambda) = 0$

or $\lambda = 2$

$\therefore$ $\vec{b_1} = 4\hat{i} - 2\hat{j} - 4\hat{k}$ ½

and $\vec{b_2} = 3\hat{i} + 4\hat{j} + \hat{k}$ ½

or $(7\hat{i} + 2\hat{j} - 3\hat{k}) = (4\hat{i} - 2\hat{j} - 4\hat{k}) + (3\hat{i} + 4\hat{j} + \hat{k})$ ½

[CBSE Marking Scheme 2017 (Modified)]

Q. 8. Show that the points *A*, *B*, *C* with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ respectively, are the vertices of a right-angled triangle, hence find the area of the triangle.

R&U [O.D. Set-I, 2017]

Sol. $\vec{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}$, $\vec{BC} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$ **1**

Since $\vec{AB}, \vec{BC}, \vec{CA}$ are not parallel vectors, and

$\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$ $\therefore$ *A*, *B*, *C* form a triangle

Also $\vec{BC} \cdot \vec{CA} = 0$ $\therefore$ *A*, *B*, *C* form a right triangle **1**

Area of $\Delta = \frac{1}{2}|\vec{CA} \times \vec{BC}| = \frac{1}{2}\sqrt{210}$ unit^2 **1**

[CBSE Marking Scheme 2017 (Modified)]

Topper Answer, 2017

$\vec{OA} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$

$\vec{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

$\vec{OA} = 2\hat{i} - \hat{j} + \hat{k}$
$\vec{OB} = \hat{i} - 3\hat{j} - 5\hat{k}$
$\vec{OC} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

36

$\vec{BC} = \vec{OC} - \vec{OB} = 2\hat{i} - \hat{j} + \hat{k}$ $\quad |\vec{BC}| = \sqrt{(2)^2+(-1)^2+(1)^2} = \sqrt{6}$ units

$\vec{AC} \cdot \vec{BC} = 2 + 3 - 5 = 0$. Hence $\vec{AC} \perp \vec{BC}$ Hence $\angle C = 90°$

$|\vec{BC}|^2 + |\vec{AC}|^2 = |\vec{AB}|^2$

Hence ~~basic~~ ΔABC is right angled at C.

area $= \frac{1}{2} |\vec{AC} \times \vec{BC}|$

$= \frac{1}{2} \left\| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -5 \\ 2 & -1 & 1 \end{vmatrix} \right\| = \frac{1}{2} | \hat{i}(-8) - \hat{j}(11) + \hat{k}(5) |$

$= \frac{1}{2} \sqrt{(-8)^2 + (-11)^2 + (5)^2} = \cancel{\frac{1}{2}\sqrt{209}}$ ~~sq units~~

$= \frac{1}{2}\sqrt{210}$ sq. units

$= \sqrt{\frac{210}{4}} = \sqrt{52.5}$ sq. units.

COMPETENCY BASED QUESTIONS

Case based MCQs

I. Read the following text and answer the following questions on the basis of the same:

Solar Panels have to be installed carefully so that the tilt of the roof, and the direction to the Sun, produce the largest possible electrical power in the solar panels.

A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture , suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters $P_1(6, 8, 4)$, $P_2(21, 8, 4)$, $P_3(21, 16, 10)$ and $P_4(6, 16, 10)$ [CBSE QB-2021]

Q. 1. What are the components to the two edge vectors defined by $\vec{A}$ = PV of P_2 – PV of P_1 and $\vec{B}$ = PV of P_4 – PV of P_1? (where PV stands for position vector)

(A) 0, 0, 15 : 0, 8, 6 **(B)** 15, 0, 0 : 0, 8, 6
(C) 0, 8, 6 : 0, 0, 15 **(D)** 15, 0, 0 : 6, 8, 8

Ans. Option (B) is correct.

Explanation:

$$\vec{A} = PV \text{ of } P_2 - PV \text{ of } P_1$$
$$= 21\hat{i} + 8\hat{j} + 4\hat{k} - (6\hat{i} + 8\hat{k} + 4\hat{k})$$
$$= 15\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\vec{B} = PV \text{ of } P_4 - PV \text{ of } P_1$$
$$= 6\hat{i} + 16\hat{j} + 10\hat{k} - (6\hat{i} + 8\hat{j} + 4\hat{k})$$
$$= (0\hat{i} + 8\hat{j} + 6\hat{k})$$

$\therefore$ $A(15,0,0)$ and $B(0,8,6)$

Q. 2. What will be the standard notation with $\hat{i}$, $\hat{j}$ and $\hat{k}$ (where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along the three axes) of the vectors obtained in Q.1.

(A) $15\hat{i}+0\hat{j}+0\hat{k}, 0\hat{i}+8\hat{j}+6\hat{k}$

(B) $0\hat{i}+6\hat{j}+8\hat{k}, 15\hat{i}+0\hat{j}+0\hat{k}$

(C) $0\hat{i}+0\hat{j}+0\hat{k}, 0\hat{i}+8\hat{j}+6\hat{k}$

(D) $15\hat{i}+0\hat{j}+0\hat{k}, 6\hat{i}+8\hat{i}+0\hat{k}$

Ans. Option (A) is correct.

Q. 3. What are the magnitudes of the vectors $\vec{A}$ and $\vec{B}$?

(A) 9, 10 **(B)** 115, 50

(C) 225, 100 **(D)** 100, 200

Ans. Option (C) is correct.

Explanation:

$$|\vec{A}| = \sqrt{(15)^2+0^2+0^2}$$
$$= 225$$
$$|\vec{B}| = \sqrt{0^2+8^2+6^2}$$
$$= \sqrt{64+36}$$
$$= \sqrt{100}$$
$$= 10$$

Q. 4. What are the components to the vector $\vec{N}$, perpendicular to $\vec{A}$ and $\vec{B}$ and the surface of the roof?

(A) –90, 90 **(B)** 120, 18

(C) –90, 100 **(D)** –90, 120

Ans. Option (D) is correct.

Explanation:

$$\vec{N} = \vec{A}\times\vec{B}$$
$$N = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 15 & 0 & 0 \\ 0 & 8 & 6 \end{vmatrix}$$
$$= -15(6\hat{j}-8\hat{k})$$
$$= -90\hat{j}+120\hat{k};$$

Q. 5. What is the magnitude of $\vec{N}$?

(A) 100 **(B)** 150

(C) 50 **(D)** 90

Ans. Option (B) is correct.

Explanation:

$$\vec{N} = -90\hat{j}+120\hat{k}$$
$$|\vec{N}| = \left|\sqrt{(90)^2-(120)^2}\right|$$
$$= \left|\sqrt{8100+14400}\right|$$
$$= \left|\sqrt{22500}\right|$$
$$= 150$$

II. Read the following text and answer the following questions on the basis of the same:

A class XII student appearing for a competitive examination was asked to attempt the following questions.

Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be three non zero vectors.

[CBSE QB 2021]

Q. 1. If $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ then

(A) $\vec{a} \perp \vec{b}$ **(B)** $\vec{a} \parallel \vec{b}$

(C) $\vec{a} = \vec{b}$ **(D)** None of these

Ans. Option (A) is correct.

Q. 2. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ be the angle between them then $|\vec{a}-\vec{b}|$ is

(A) $\sin\frac{\theta}{2}$ **(B)** $2\sin\frac{\theta}{2}$

(C) $2\cos\frac{\theta}{2}$ **(D)** $\cos\frac{\theta}{2}$

Ans. Option (B) is correct.

Explanation:

$$|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b}).(\vec{a}-\vec{b})$$
$$= |\vec{a}|^2+|\vec{b}|^2 - 2\vec{a}.\vec{b}\cos\theta$$
$$= 1+1-2(1)(1)\cos\theta$$
$$\Rightarrow \quad |\vec{a}-\vec{b}|^2 = 2-2\cos\theta$$
$$= 2(1-\cos\theta)$$
$$\Rightarrow \quad |\vec{a}-\vec{b}|^2 = 2\left(2\sin^2\frac{\theta}{2}\right)$$
$$\Rightarrow \quad |\vec{a}-\vec{b}|^2 = 4\sin^2\frac{\theta}{2}$$
$$|\vec{a}-\vec{b}| = 2\sin\frac{\theta}{2}$$

Q. 3. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$ then $\vec{a}$ =

(A) $2(\vec{b}\times\vec{c})$ **(B)** $-2(\vec{b}\times\vec{c})$

(C) $\pm 2(\vec{b}\times\vec{c})$ **(D)** $2(\vec{b}\pm\vec{c})$

Ans. Option (C) is correct.

Q. 4. The area of the parallelogram formed by $\vec{a}$ and $\vec{b}$ as diagonals is

(A) 70 **(B)** 35

(C) $\frac{\sqrt{70}}{2}$ **(D)** $\sqrt{70}$

Ans. Option (C) is correct.

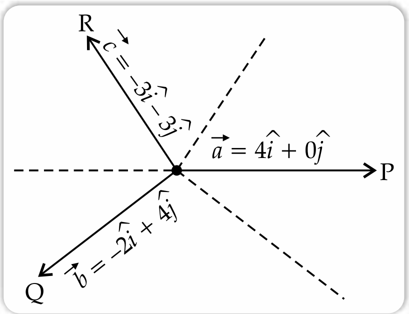
Case based Subjective Questions (2 marks each)

I. Read the following text and answer the following questions on the basis of the same:

Team P, Q, R went for playing a tug of war game. Teams P, Q, R have attached a rope to a metal ring and is trying to pull the ring into their own areas (team areas when in the given figure below). Team P pulls with force $F_1 = 4\hat{i} + 0\hat{j}$ KN

Team Q pulls with force $F_2 = -2\hat{i} + 4\hat{j}$ KN

Team R pulls with force $F_3 = -3\hat{i} - 3\hat{j}$ KN



Q. 1. What is the magnitude of the teams combined force? Also, find which team will win the game?

Sol. Let F be the combined force,

$$\therefore \quad \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$
$$= (4\hat{i} + 0\hat{j}) + (-2\hat{i} + 4\hat{j}) + (-3\hat{i} - 3\hat{j})$$
$$= (4 - 2 - 3)\hat{i} + (0 + 4 - 3)\hat{j}$$
$$= -\hat{i} + \hat{j}$$
$$\therefore \quad |\vec{F}| = \left|\sqrt{(-1)^2 + 1^2}\right|$$
$$= \left|\sqrt{2}\right| \text{ KN} \qquad \mathbf{1}$$

Now, $|\vec{F}_1| = \left|\sqrt{(4)^2 + 0^2}\right| = 4$ KN

$|\vec{F}_2| = \left|\sqrt{(-2)^2 + 4^2}\right| = \sqrt{20}$ KN

and $|\vec{F}_3| = \left|\sqrt{(-3)^2 + (-3)^2}\right| = \sqrt{18}$ KN

Here, magnitude of force F_2 is greater, therefore team Q will win the game. **1**

Q. 2. In what direction is the ring getting pulled ?

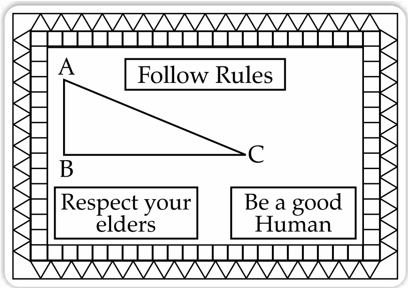
Sol. We have,

Combined force, $\vec{F} = -\hat{i} + \hat{j}$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{1}{-1}\right) \qquad \mathbf{1}$$
$$= \tan^{-1}(1)$$
$$= \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$$
$$= \frac{3\pi}{4} \text{ radians} \qquad \mathbf{1}$$

II. Read the following text and answer the question on the basis of the same.

The slogans on chart papers are to be place on a school bulletin board at the points A, B and C displaying A (follow Rules), B (Respect your elders) and C (Be a good human). The coordinates of these points are (1, 4, 2), (3, –3, –2) and (–2, 2, 6), respectively.



Q. 1. If $\vec{a}, \vec{b}$ and $\vec{c}$ be the position vectors of points A, B, C, respectively, then find $|\vec{a} + \vec{b} + \vec{c}|$.

Sol. Here,

Position vector of A is $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$

Position vector of B is $\vec{b} = 3\hat{i} - 3\hat{j} - 2\hat{k}$

Position vector of C is $\vec{c} = -2\hat{i} + 2\hat{j} + 6\hat{k}$ **1**

$$\therefore \quad \vec{a} + \vec{b} + \vec{c} = (1 + 3 - 2)\hat{i} + (4 - 3 + 2)\hat{j} + (2 - 2 + 6)\hat{k}$$
$$= 2\hat{i} + 3\hat{j} + 6\hat{k} \qquad \tfrac{1}{2}$$

Thus, $|\vec{a} + \vec{b} + \vec{c}| = \left|\sqrt{(2)^2 + (3)^2 + (6)^2}\right|$

$$= \left|\sqrt{4 + 9 + 16}\right| \qquad \tfrac{1}{2}$$
$$= \sqrt{29}$$

Q. 2. Find area of ΔABC.

Sol. We have, Δ(1, 4, 2), B(3, –3, –2) and C(–2, 2, 6)

Now, $\overrightarrow{AB} = \vec{b} - \vec{a} = 2\hat{i} - 7\hat{j} - 4\hat{k}$

and $\overrightarrow{AC} = \vec{c} - \vec{a} = -3\hat{i} - 2\hat{j} + 4\hat{k}$

$$\therefore \quad \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & -4 \\ -3 & -2 & 4 \end{vmatrix}$$
$$= \hat{i}(-28 - 8) - \hat{j}(8 - 12) + \hat{k}(-4 - 21)$$
$$= -36\hat{i} + 4\hat{j} - 25\hat{k} \qquad \mathbf{1}$$

Now, $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-36)^2 + 4^2 + (-25)^2}$

$$= \left|\sqrt{1296 + 16 + 625}\right| = \sqrt{1937}$$

$$\therefore \quad \text{Area of } \Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$$
$$= \frac{1}{2}\sqrt{1937} \text{ sq. units} \qquad \mathbf{1}$$

Solutions for Practice Questions (Topic-1)

Very Short Answer Type Questions

2. $\hat{j}$ **1**

[CBSE SQP Marking Scheme, 2020-21]

Commonly Made Error

- Some students find any vector instead of unit vector. Some others find the unit vector in the same direction.

Answering Tip

- Read the question carefully and practice more problems involving unit vectors.

3. $\vec{a} = \hat{i}, \vec{b} = \hat{j}$ [or any other correct answer] **1**

[CBSE Marking Scheme, 2017-18]

Short Answer Type Questions-I

2.
$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ 2 & -1 & 2 \end{vmatrix}$$

$$= -2\hat{i}+4\hat{j}+4\hat{k} \qquad \mathbf{1}$$

$$|\vec{a}\times\vec{b}| = |\sqrt{4+16+16}| = 6 \qquad ½$$

Area of the parallelogram $= \dfrac{|\vec{a}\times\vec{b}|}{2} = 3$ sq. units.½

[CBSE Marking Scheme 2018-19]

Commonly Made Error

- Mostly students use the formula to find the area of parallelogram when sides are given.

Answering Tip

- Clarify the concept of finding area of parallelogram whose diagonal are vectors.

3. The angle θ between the vectors $\vec{a}$ and $\vec{b}$ is given by

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} \qquad \mathbf{1}$$

i.e., cosθ

$$= \frac{(\hat{i}+\hat{j}-\hat{k}).(\hat{i}-\hat{j}+\hat{k})}{\sqrt{(1)^2+(1)^2+(-1)^2}.\sqrt{(1)^2+(-1)^2+(1)^2}}$$

i.e., $\cos\theta = \dfrac{1-1-1}{\sqrt{3}.\sqrt{3}}$

i.e., $\cos\theta = -\dfrac{1}{3} \Rightarrow \theta = \cos^{-1}\left(-\dfrac{1}{3}\right).$ **1**

[CBSE Marking Scheme 2018-19]

Solutions for Practice Questions (Topic-2)

Very Short Answer Type Questions

2. $|\vec{a}| = |\vec{b}| = 3$ ½+½

[CBSE Marking Scheme, 2018]

OR

Topper Answer, 2018

Sol.

$|\vec{a}| = |\vec{b}|$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}| \cos\theta$

$\frac{9}{2} = |\vec{a}||\vec{a}| \cos 60^\circ$

$\frac{9}{2} = |\vec{a}|^2 \times \frac{1}{2}$

$9 = |\vec{a}|^2$

$|\vec{a}| = 3.$

Also $|\vec{a}| = |\vec{b}|$

$|\vec{b}| = 3$

$|\vec{a}| = |\vec{b}| = 3$ Ans

5. $(2\hat{i}+3\hat{j}+2\hat{k})\cdot(2\hat{i}+2\hat{j}+\hat{k}) = 12$ ½

$$p = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} \quad \text{or} \quad p = \frac{12}{|\vec{b}|}$$

$$= \frac{12}{3} = 4 \qquad ½$$

[CBSE Marking Scheme 2015]

Short Answer Type Questions-I

2. $$(\vec{a}+\vec{b})(\vec{a}-\vec{b}) = 12$$
$$\Rightarrow |\vec{a}|^2-|\vec{b}|^2 = 12$$
$$\Rightarrow 3|\vec{b}|^2 = 12$$
$$\Rightarrow |\vec{b}|^2 = 4 \quad 1$$
Now, $$|\vec{a}|^2 = 12+|\vec{b}|^2$$
$$= 16$$
$$|\vec{a}| = 4 \quad 1$$

[CBSE Marking Scheme 2020]

Detailed Solution:

$$|\vec{a}| = ?$$
$$|\vec{b}| = ?$$
Given, $$|\vec{a}| = 2|\vec{b}|$$
and $$(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 12$$
$$\Rightarrow \vec{a}.\vec{a}-\vec{a}.\vec{b}+\vec{a}.\vec{b}-\vec{b}.\vec{b} = 12$$
$$\Rightarrow \vec{a}.\vec{a}-\vec{b}.\vec{b} = 12$$
$$\Rightarrow |\vec{a}|^2-|\vec{b}|^2 = 12$$
$$\Rightarrow (2|\vec{b}|)^2-(|\vec{b}|)^2 = 12$$
$$\Rightarrow 4|\vec{b}|^2-|\vec{b}|^2 = 12$$
$$\Rightarrow 3|\vec{b}|^2 = 12$$
$$\Rightarrow |\vec{b}|^2 = 4$$
$$\Rightarrow |\vec{b}| = 2$$
$$\Rightarrow |\vec{a}| = 2|\vec{b}| = 2(2) = 4$$

Hence, $|\vec{a}| = 4$ and $|\vec{b}| = 2$

5. $\vec{a} = 2\hat{i}-\hat{j}+\hat{k}, \vec{b} = \hat{i}-2\hat{j}+\hat{k}, \vec{a}.\vec{b} = 5, |\vec{b}| = \sqrt{6}.$ ½

The required projection (vector) of $\vec{a}$ on $\vec{b}$

$$= \frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\vec{b} \quad 1$$

$$= \frac{5}{6}(\hat{i}-2\hat{j}+\hat{k}). \quad ½$$

[CBSE Marking Scheme, 2017]

Short Answer Type Questions-II

3. $\overrightarrow{AB} = \hat{i}+4\hat{j}-\hat{k}$ ½

$\overrightarrow{CD} = -2\hat{i}-8\hat{j}+2\hat{k}$ ½

Let required angle be θ.

Then $\cos\theta = \dfrac{\overrightarrow{AB}\cdot\overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|} = \dfrac{-2-32-2}{\sqrt{18}\sqrt{72}} = -1$ 1

$\Rightarrow \theta = 180°$ or π ½

Since $\theta = \pi$ so $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear. ½

[CBSE Marking Scheme, 2019] (Modified)

Solutions for Practice Questions (Topic-3)

Very Short Answer Type Questions

1. $\frac{1}{2}|2\hat{i}\times(-3\hat{j})| = \frac{1}{2}|-6\hat{k}| = 3$ sq. units 1

[CBSE SQP Marking Scheme 2020-21]

2. Angle between $\vec{a}\times\vec{b}$ and $\vec{b}\times\vec{a}$ is π. 1

[CBSE Marking Scheme 2017]

Short Answer Type Questions-I

3. $$\vec{a}\times\vec{b} = 7\hat{i}-6\hat{j}-10\hat{k} \quad 1$$
and $$|\vec{a}\times\vec{b}| = \sqrt{185} \quad ½$$
Required unit vector
$$= \frac{1}{\sqrt{185}}(7\hat{i}-6\hat{j}-10\hat{k}) \quad ½$$

[CBSE Marking Scheme 2020]

Detailed Solution:

Given, $$\vec{a} = 4\hat{i}+3\hat{j}+\hat{k}$$
and $$\vec{b} = 2\hat{i}-\hat{j}+2\hat{k}$$
Hence, $$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 4 & 3 & 1 \\ 2 & -1 & 2 \end{vmatrix}$$
$$= \hat{i}(6+1)-\hat{j}(8-2)+\hat{k}(-4-6)$$
$$= 7\hat{i}-6\hat{j}-10\hat{k}$$

Unit vector perpendicular to each of the vector $\vec{a}$ and $\vec{b}$

$$= \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{7\hat{i}-6\hat{j}-10\hat{k}}{\sqrt{(7)^2+(-6)^2+(-10)^2}}$$
$$= \frac{7}{\sqrt{185}}\hat{i}-\frac{6}{\sqrt{185}}\hat{j}-\frac{10}{\sqrt{185}}\hat{k}$$

Commonly Made Error

- Mostly students get confused between the formula for areas of triangle and parallelogram.

Answering Tips

- Practice problems based on area.

4. Let θ be the angle between $\vec{a}$ and $\vec{b}$, then

$$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|} = \frac{7}{2.7} = \frac{1}{2} \quad 1\tfrac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \quad \tfrac{1}{2}$$

[CBSE Marking Scheme, 2019]

Detailed Solution :

Given, $|\vec{a}| = 2, |\vec{b}| = 7$ and $\vec{a}\times\vec{b} = 3\hat{i}+2\hat{j}+6\hat{k}$

Let 'θ' be the angle between $\vec{a}$ and $\vec{b}$.

Since, $\sin\theta = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

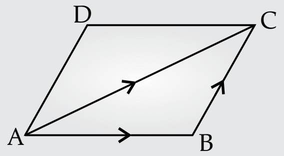
$\Rightarrow \sin\theta = \dfrac{\sqrt{(3)^2+(2)^2+(6)^2}}{2\times 7}$

$\Rightarrow \sin\theta = \dfrac{1}{2}$ or $\theta = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$

Short Answer Type Questions-II

6. $\vec{BC} = \vec{AC} - \vec{AB} = \hat{i} - \hat{j} + \hat{k}$ **1**

Area $= |\vec{AB}\times\vec{BC}|$



$= \text{magnitude of } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$ **1**

$= |5\hat{i} + \hat{j} - 4\hat{k}|$

$= \sqrt{42}$ sq. units **1**

[CBSE Marking Scheme 2017 (Modified)]

9. Here $3\vec{a} + 2\vec{b} = 5\hat{i}+7\hat{j}+9\hat{k}$

and $3\vec{a} - 2\vec{b} = \hat{i}-\hat{j}-3\hat{k}$ **1**

Let $\vec{c}$ be the vector perpendicular to both $(3\vec{a}+2\vec{b})$ & $(3\vec{a}-2\vec{b})$.

Then, $\vec{c} = (3\vec{a}+2\vec{b})\times(3\vec{a}-2\vec{b})$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 7 & 9 \\ 1 & -1 & -3 \end{vmatrix}$

$= -12\hat{i} + 24\hat{j} - 12\hat{k}$ **2**

[CBSE Marking Scheme 2016 (Modified)]

Detailed Solution :

Given $\vec{a} = \hat{i}+\hat{j}+\hat{k}$

$\vec{b} = \hat{i}+2\hat{j}+3\hat{k}$

$3\vec{a} = 3\hat{i}+3\hat{j}+3\hat{k}$

and $2\vec{b} = 2\hat{i}+4\hat{j}+6\hat{k}$

Let $\vec{c}$ be the vector perpendicular to both $(3\vec{a}+2\vec{b})$ and $(3\vec{a}-2\vec{b})$.

$\therefore \quad 3\vec{a}+2\vec{b} = 5\hat{i}+7\hat{j}+9\hat{k}$

and $3\vec{a}-2\vec{b} = \hat{i}-\hat{j}-3\hat{k}$

$\therefore \quad \vec{c} = (3\vec{a}+2\vec{b})\times(3\vec{a}-2\vec{b})$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 7 & 9 \\ 1 & -1 & -3 \end{vmatrix}$

$= \hat{i}(-21+9) - \hat{j}(-15-9) + \hat{i}(-5-7)$

$= -12\hat{i} + 24\hat{j} - 12\hat{k}$

REFLECTIONS

- Will you be able to handle the complex situation related to direction cosines and direction ratio?
- Can you early apply the dot and cross product of vectors in Mechanical engineering.?

❑❑❑